AD-773 801

QUANTO - A CODE TO OPTIMIZE WEAPON ALLOCATIONS

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January 1974

UNCLASSIFIED
Security Classification

AD 773 801

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Air Force Weapons Laboratory (SAB) Kirtland Air Force Base, New Mexico 87117	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED
	2b. GROUP

3. REPORT TITLE

QUANTO--A CODE TO OPTIMIZE WEAPON ALLOCATIONS

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
1 September 1971 through 1 October 1973

5. AUTHOR(S) (First name, middle initial, last name)

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6. REPORT DATE January 1974	7a. TOTAL NO. OF PAGES 180	7b. NO. OF REFS 4

8a. CONTRACT OR GRANT NO	9a. ORIGINATOR'S REPORT NUMBER(S) AFWL-TR-73-242
b. PROJECT NO 8809	
c. Task 04	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY AFWL (SAB) Kirtland AFB, NM 87117

13. ABSTRACT

(Distribution Limitation Statement A)

An advanced computer model has been developed within the Air Force Weapons Laboratory (AFWL) to study the effects of a sea-launched ballistic missile (SLBM) attack on targets consisting of a flushing aircraft force. Using the technique of Lagrangian multiplier optimization, a near-optimal allocation of SLBMs to targets is produced. Although a number of other computer codes exist which model the same situation, the AFWL code is believed to be the only one possessing all of the following features: (1) the determination of the SLBM laydown against a mixed aircraft force (for instance, bombers and tankers); (2) the automatic consideration of differing aircraft hardnesses, flyout profiles, level-off altitudes, and kill values; (3) the treatment of an SLBM attack by multiple types of SLBMs, accounting for differing missile trajectories, yields, launch intervals, reliabilities, and numbers of missiles per submarine; (4) automated relocation of the submarines and/or the aircraft, if desired, to enhance the goals of each; and (5) thermal as well as overpressure kills, accounting for such additional nuclear effects parameters as detonation altitude, haze level, water vapor, visibility, and albedo, through the use of the most current advanced nuclear effects codes developed by AFWL.

179

DD FORM 1473

UNCLASSIFIED
Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Flush Survivability SLBM Allocation Laydown Beddown Lagrange Optimization Nuclear						

AFWL-TR-73-242

QUANTO--A CODE TO OPTIMIZE WEAPON ALLOCATIONS

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Final Report for Period 1 September 1971 through 1 October 1973

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FOREWORD

The research was performed under Program Element 62601F, Project 8809, Task 04.

Inclusive dates of research were 1 September 1971 through 1 October 1973. The report was submitted 25 October 1973 by the Air Force Weapons Laboratory Project Officer, Major Arthur R. Geldbach (SAB).

The advanced computer model QUANTO has been developed within the Air Force Weapons Laboratory to study various scenarios involving sea-launched ballistic missile attacks on bomber air bases. The QUANTO model has been reviewed by interested Air Staff agencies, the Air Force Systems Command, and the Strategic Air Command, and is considered appropriate for use in activities relating to bomber force prelaunch survival. However, prudence should be exercised in its use, because of its sensitivity and the dynamic nature of the problem.

The basic model was developed by Major Richard Conway. A large portion of the debugging and exercising of the code was done by Mr. Eugene Omoda and Mr. William Peay. The assistance of Mr. Harry Murphy in utilizing the operating system and remote terminal, and that of Mr. Al Sharp in incorporating the thermal and overpressure routines into QUANTO were also invaluable in the development process.

This technical report has been reviewed and is approved.

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ABBREVIATIONS AND SYMBOLS

P_k	Probability of destroying the target or any single aircraft at the target, given that the weapon successfully detonates
p	Number of candidate submarine locations
A_a	Candidate location a (a = 1, 2, ..., p) of attacking submarine(s)
S_a	Number of missiles on all submarines at point A_a
t_a	Number of submarines at point A_a
T_i	Target i consisting of aircraft with value V_i
V_i	Value of the aircraft on target i
L	Number of weapon groups
n_{ij}	Number of weapons on target i from a weapon in group j, j = 1, 2, ..., L
$f(n_{ij})$	The objective function to be maximized
M	Number of targets with values V_i
$S_{ij}^{n_{ij}}$	Probability of survival of aircraft on target i from n_{ij} weapons in weapon group j
N_j	Number of weapons in group j
λ_j	Constants (the Lagrange Multipliers)
$h(n_{ij},\lambda_j)$	The Lagrangian function
λ_{ij}	Variables dependent on n_{kj} (k = 1, 2, ..., M) which assist in determining the Lagrange multipliers λ_j and the optimal laydown n_{ij}
Δn	The number of weapons shifted with each iteration in the convergence to the optimal laydown
$r(\Delta n)$	A function representing the kill contribution to the objective function $f(n_{ij})$ from targets k and m after Δn weapons are moved from target k to target m.
$\lambda_{k\ell}$	$\min_i \left\{\lambda_{i\ell} \text{ such that } n_{i\ell} \geq 0.0001\right\}$, i = 1, 2, ..., M

ABBREVIATIONS AND SYMBOLS (cont'd)

$\lambda_{m\ell}$	$\max_i \left\{\lambda_{i\ell}\right\}$, i = 1, 2, ..., M
ε	The tolerance used to test for convergence of the λ_{ij}'s in obtaining the optimal laydown n_{ij}
s	The number of salvos of SLBMs on a submarine which is a candidate for relocation
R_1	The radial distance of the most distant aircraft from the centroid at the time of a given weapon arrival
R_N	The radial distance of the least distant aircraft from the centroid at the time of a given weapon arrival
A_L[for x]	The circular lethal area when the detonation point is at distance x from the centroid
A_{LAN}[for x]	The lethal area occupied by aircraft in the annulus with radii R_1 and R_N when the detonation point is at distance x from the centroid
R_{LR}[for x]	The distance from detonation point to lethal region boundary, in a direction away from the centroid
V_{iB}	Total bomber value on base i
V_{iT}	Total tanker value on base i
S_{ijB}	Survival probability of bombers on target i from one weapon in weapon group j
S_{ijT}	Survival probability of tankers on target i from one weapon in weapon group j
$P_{k/B}$	Probability of destroying bombers at a target, given that the weapon successfully detonates at the target
$P_{k/T}$	Probability of destroying tankers at a target, given that the weapon successfully detonates at the target
$R_{LR/MAX}$	The distance from detonation point to the farthest lethal region boundary (for all aircraft types), in a direction away from the centroid
ℓn	Napierian base logarithm

SECTION I

INTRODUCTION

The theory of the allocation of the sea-launched ballistic missiles (SLBM) against a force of aircraft flushing from their respective airbases and the defensive reactions to given threat levels is discussed. The analysis, which led to the models discussed later, has culminated in a computer program called QUANTO. The model used in QUANTO has as its inputs latitude and longitude coordinates of target and submarine locations, aircraft beddowns, aircraft and missile flight parameters, and aircraft vulnerability levels. Consequently, the code is useful for studying the effects of variations in a number of parameters.

QUANTO analyzes three types of problems important to strategic planners:

Case I: Given specific locations (A_a) for a fixed number of attacking submarines and a specific beddown for aircraft at locations T_i, QUANTO can compute where the assigned missiles from A_a should go.

Case II: Given specific beddown for aircraft at locations T_i, QUANTO can optimize the locations for the submarines among a set of candidate locations A_a.

Case III: Given specific submarine locations A_a, QUANTO can optimize the beddown of aircraft at T_i.

Lagrange multipliers are used in the optimization procedures of QUANTO. A brief review of this technique is presented in appendix I and is intended to acquaint the reader with the basic mathematics involved.

The QUANTO code has been developed within the Air Force Weapons Laboratory (AFWL). It was intended originally as a vehicle for increasing the understanding of the operation of a computer program called COG, which dealt only with Case I (as of May 1971), that was written by the Lambda Corporation (ref. 1). Compared to other codes, QUANTO permits a more detailed and accurate analysis, because weapons and their detonations are handled individually, rather than as members of fixed weapon patterns. Studies show that QUANTO produces a considerably better

allocation than does COG. Further investigation, substantiated by simulation of the attack through the use of another AFWL code, supports the assumptions and models used in QUANTO. Hence, QUANTO provides a means for comparing and evaluating the effectiveness of other weapon allocation codes. More importantly, QUANTO provides a framework for modification and extension in further studies of total bomber/tanker force survivability.

SECTION II

BASIC WEAPON ALLOCATION PROBLEM

In this study, the attacking force of submarines (figure 1) is distributed among points A_a, a = 1, 2, ..., p, where submarines at A_a each carry s_a SLBM weapons. At the same time, suppose that the targets, T_i, i = 1, 2, ..., M, have values V_i. If one were to visualize this engagement as in figure 1, it becomes apparent that many strategies are open to the attacher and defender. For example, the attacking force could put all missiles on target T_1. On the other hand, the missiles could be distributed among all targets. As for the defender, he could place his bombers and tankers throughout the target areas evenly or perhaps all on the same base. The multiplicity of possibilities increases with each new missile or aircraft, making hand calculations impractical. The approach taken to solve this problem is to use the method of Lagrange-Multipliers to produce a near-optimal allocation of SLBMs to targets consisting of escaping aircraft. To construct the objective function which describes the expected value killed, one must first develop the survival probability S_j for each weapon. This figure is the probability that a single aircraft, given an escaping time-dependent pattern of aircraft taking off from the airfield, survives one incoming SLBM. The probability of kill is then

$$P_k = 1 - S_j \tag{1}$$

Suppose now that n weapons are delivered to a target, and the survival probabilities S_j of the target from each weapon, j = 1, 2, ..., n, are independent; then the probability of destroying the target is

$$P_k = 1 - \prod_{j=1}^{n} S_j \tag{2}$$

For a system of M targets, each having a value V_i, the expected return from delivery of all weapons is

A_0 = THE ATTACKING FORCE OF SUBMARINES (COULD BE ZERO OR MORE THAN ONE AT EACH LOCATION).

T_i = TARGETS

V_i = VALUE OF TARGETS T_i

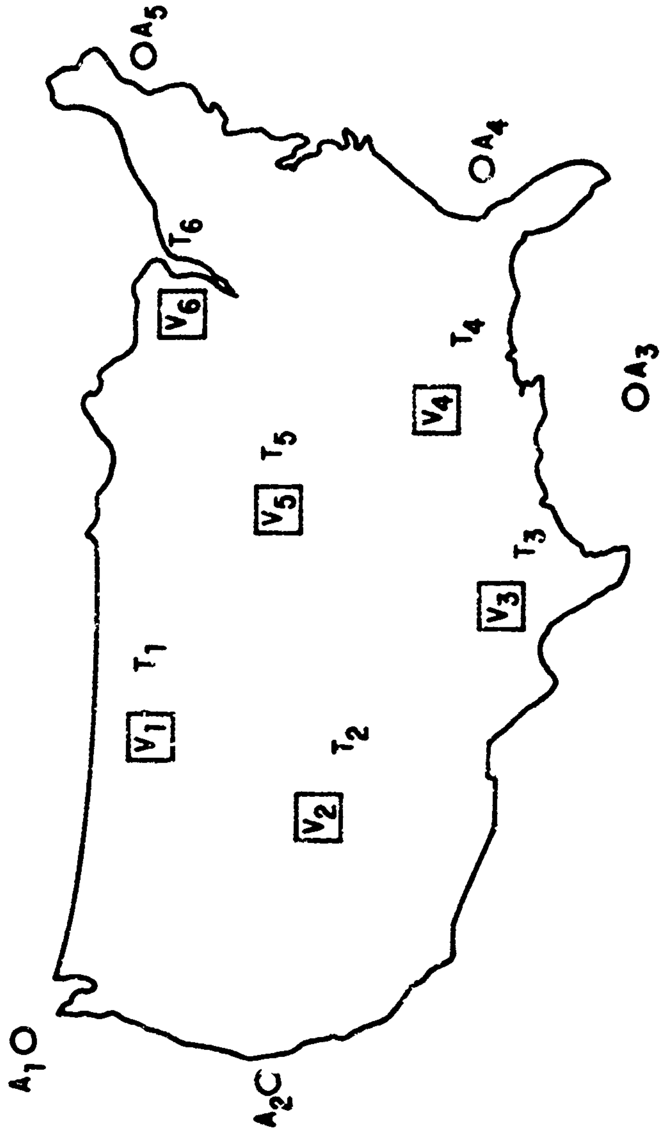

Figure 1. Attacker versus Attacked Forces

$$\sum_{i=1}^{M} V_i \left\{1 - \prod_{j} S_{ij}\right\} \tag{3}$$

where S_{ij} is the probability of survival of target i from weapon j. The product in expression (3) for target i includes only those survivabilities corresponding to the weapons which are aimed at target i. In practice, there are weapon "groups," where the weapons in each group are so nearly identical in characteristics and location that no distinction between them is required for purposes of allocation. Hence, in practice, S_{ij} is raised to the n_{ij} power, where n_{ij} is the number of weapons from weapon group j that are targeted against target i. It is easy to see that the product

$$\prod_{j} S_{ij}^{\,n_{ij}}$$

is not changed if the weapon groups j are included for which $n_{ij} = 0$. Consequently, if L is the total number of weapon groups, the expected aircraft kill may be written

$$\sum_{i=1}^{M} V_i \left[1 - \prod_{j=1}^{L} S_{ij}^{\,n_{ij}}\right] \tag{4}$$

Table I clarifies the submarine input parameters used in QUANTO. The table has six columns, the first of which is submarine locations, given to QUANTO in terms of latitude and longitude coordinates, surrounding a given target country. The second and fifth columns contain the same information and are presented separately to emphasize the fact that the number of submarines and number of missiles per group are the same since all the submarines at a given location are assumed to fire a missile apiece at the same time. Note here that zero submarines are allowed at a given location. In column four the numbering 1 to 4 is applied to two types of weapons, each of which is restricted to either the Atlantic or Pacific Ocean. Numbers 1 and 2 may identify weapon types 1 and 2 in the Pacific, whereas 3 and 4 may represent weapon types 1 and 2 in the Atlantic. Since submarines may be shifted only among locations which have like missile type identifiers in QUANTO, such a numbering system prevents submarines from relocating to a different ocean. The last column is the numbering given to the

Table I

SUBMARINE INPUT DATA TO QUANTO (EXAMPLE)

Location	Number Submarines	Number Missiles/ Sub (L = 42)	Type Missile	Number Missiles/ Group (N_j)	Weapon Groups (j)
1	1	8	1	1	1 - 8
2	2	6	2	2	9 - 14
3	0	8	1	0	15 - 22
4	4	6	2	4	23 - 28
5	1	8	3	1	29 - 36
6	3	6	4	3	37 - 42

weapon groups. Note here that the numbering in row 2 goes from 9 through 14. Each weapon group here consists of two missiles in the same salvo since there are two submarines at this location. Missiles may be placed in the same group if they have identical trajectories and are launched at the same time from the same point. Also, row 3 allows for a set of weapon groups even though no submarines are initially placed at submarine location 3 (although there may be subsequently, if the submarine-placement optimizer of Case II is exercised).

The basic allocation problem is to maximize the expected kill given by expression (4) by sending the missiles to the proper targets. Since the allocation of missiles to targets is expressed by the values n_{ij}, the problem is to find the integer values n_{ij} which result in the greatest kill while satisfying constraints on the number of weapons available in each group.

SECTION III

USE OF LAGRANGE MULTIPLIERS IN THE QUANTO COMPUTER CODE

1. FORMULATION FOR SOLUTION

The weapon allocation problem is one of determining the optimal allocations n_{ij} of weapons to targets to maximize the expected kill value

$$f\left(n_{ij}\right) = \sum_{i=1}^{M} V_i \left[1 - \prod_{j=1}^{L} S_{ij}^{\,n_{ij}}\right] \tag{5}$$

subject to the stockpile constraints

$$\sum_{i=1}^{M} n_{ij} = N_j, \quad j = 1, 2, \ldots, L \tag{6}$$

where N_j is the number of weapons in weapon group j. Fractional allocations in n_{ij} are permitted in the solution of this problem, but each n_{ij} must satisfy

$$0 \le n_{ij} \le N_j \tag{7}$$

As described in appendix I, this problem gives rise to the Lagrangian function

$$h\left(n_{ij},\lambda_j\right) = f\left(n_{ij}\right) + \sum_{j=1}^{L} \lambda_j \left[\sum_{i=1}^{M} \left(n_{ij}\right) - N_j\right] \tag{8}$$

In seeking the extremum of the Lagrangian function h n_{ij},λ_j , the values of $\left(n_{ij},\lambda_j\right)$ are sought which satisfy the following necessary conditions for a solution using this Lagrangian Multiplier technique

$$\frac{\partial h}{\partial n_{k\ell}} = -V_k \left(\ell n\ S_{k\ell}\right) \prod_{j=1}^{L} S_{kj}^{\,n_{kj}} + \lambda_\ell = 0 \tag{9}$$

$$k = 1, 2, \ldots, M; \ \ell = 1, 2, \ldots, L$$

If variables $\lambda_{k\ell}$ (dependent on n_{kj}, j = 1, 2, ..., L) are defined as

$$\lambda_{k\ell} \equiv -V_k \left(\ell n\ S_{k\ell}\right) \prod_{j=1}^{L} S_{kj}^{\ n_{kj}} \tag{10}$$

the system (equation (9)) of (M X L) equations becomes

$$\lambda_{k\ell} = -\lambda_\ell,\ k = 1, 2, \ldots, M;\ \ell = 1, 2, \ldots, L$$

Now fix ℓ and consider the subsystem of M equations

$$\begin{aligned} \lambda_{1\ell} &= -\lambda_\ell \\ \lambda_{2\ell} &= -\lambda_\ell \\ &\ \ \vdots \\ \lambda_{M\ell} &= -\lambda_\ell \end{aligned} \tag{11}$$

A word is in order concerning notation. In equations (11), λ_ℓ is one of the unknown Lagrange multipliers. The variables $\lambda_{k\ell}$ (k = 1, 2, ..., M) are computable if one has the values of n_{kj} (j = 1, 2, ..., L). The technique used for finding the values of λ_ℓ and n_{kj} (k = 1, 2, ..., M; j = 1, 2, ..., L) which satisfy the system (equations (11)) of M equations takes advantage of the fact that all the $\lambda_{k\ell}$ should equal the same quantity, namely $-\lambda_\ell$. The method chooses values of n_{kj} iteratively, subject to the constraints, so that the values of $\lambda_{k\ell}$ (k = 1, 2, ..., M) approach a single value, namely $-\lambda_\ell$.

2. ITERATIVE PROCEDURE

An initial allocation of weapons to targets n_{ij} is input to QUANTO, and the variables λ_{ij} are computed. Suppose for a given weapon group ℓ that $\lambda_{k\ell} < \lambda_{m\ell}$ and $n_{k\ell} \geq 0.0001$. Then by moving an appropriate number Δn of weapons in group ℓ from target k to target m, $\lambda_{k\ell}$ and $\lambda_{m\ell}$ may be made more nearly equal. Note

that $n_{k\ell}$ must be initially positive or there would be no weapons to shift. In fact, were it not for the restriction that $n_{k\ell}$ may not be reduced to a negative amount (i.e., $\Delta n \leq n_{k\ell}$), $\lambda_{k\ell}$ and $\lambda_{m\ell}$ could be made equal in all cases. The value of Δn which would make the new values of $\lambda_{k\ell}$ and $\lambda_{m\ell}$, say $\hat{\lambda}_{k\ell}$ and $\hat{\lambda}_{m\ell}$, equal is the value of Δn which satisfies

$$\hat{\lambda}_{k\ell} = V_k \left(\ell n\ S_{k\ell}\right) S_{k1}^{\ n_{k1}} S_{k2}^{\ n_{k2}} \ldots S_{k\ell}^{\ n_{k\ell}-\Delta n} \ldots S_{kL}^{\ n_{kL}}$$

$$= V_m \left(\ell n\ S_{m\ell}\right) S_{m1}^{\ n_{m1}} S_{m2}^{\ n_{m2}} \ldots S_{m\ell}^{\ n_{m\ell}+\Delta n} \ldots S_{mL}^{\ n_{mL}} = \hat{\lambda}_{m\ell}$$

This may be written as

$$\hat{\lambda}_{k\ell} = \lambda_{k\ell}\, S_{k\ell}^{\ -\Delta n} = \lambda_{m\ell}\, S_{m\ell}^{\ +\Delta n} = \hat{\lambda}_{m\ell}$$

Therefore,

$$\Delta n = \frac{\ell n \dfrac{\lambda_{k\ell}}{\lambda_{m\ell}}}{\ell n \left(S_{k\ell}\, S_{m\ell}\right)} \tag{12}$$

Since Δn is not permitted to be so large that $(n_{k\ell} - \Delta n)$ becomes negative, the actual number of weapons shifted is

$$\Delta n = \min \left\{ n_{k\ell}, \frac{\ell n \left(\lambda_{k\ell}/\lambda_{m\ell}\right)}{\ell n \left(S_{k\ell}\, S_{m\ell}\right)} \right\} \tag{13}$$

This shift of weapons gives rise to a new j_{ij} and new λ_{ij}. Repeated shifts ultimately force each pair, $\left(\lambda_{k\ell}, \lambda_{m\ell}\right)$, for each weapon group ℓ, to be equal (for those targets k and m for which weapons from group ℓ end up being allocated).

Although the restriction $\Delta n \leq n_{k\ell}$ makes it impossible to force the equality of every pair $\left(\lambda_{k\ell}, \lambda_{m\ell}\right)$, the preceding choice of Δn does result in the greatest increase in the objective function which can result from such a shift of weapons in group ℓ from target k to target m. To see this consider the function

$$r(\Delta n) = V_k \left[1 - S_{k1}^{\;n_{k1}} \cdot S_{k2}^{\;n_{k2}} \cdots S_{k\ell}^{\;n_{k\ell}-\Delta n} \cdots S_{kL}^{\;n_{kL}}\right]$$

$$+ V_m \left[1 - S_{m1}^{\;n_{m1}} \cdot S_{m2}^{\;n_{m2}} \cdots S_{m\ell}^{\;n_{m\ell}+\Delta n} \cdots S_{mL}^{\;n_{mL}}\right] \qquad (14)$$

which represents the kill contribution to the objective function $f(n_{ij})$ (equation (5)) from targets k and m after Δn weapons are moved from target k to target m. The best choice of Δn is where $r(\Delta n)$ achieves its maximum within the interval $0 \leq \Delta n \leq n_{k\ell}$. The unrestricted maximum of $r(\Delta n)$ occurs where

$$\frac{d\, r(\Delta n)}{d(\Delta n)} = 0$$

i.e.,

$$r'(\Delta n) = \lambda_{m\ell}\, S_{m\ell}^{\;+\Delta n} - \lambda_{k\ell}\, S_{k\ell}^{\;-\Delta n} = 0$$

or

$$\Delta n^* = \frac{\ell n\left(\lambda_{k\ell}/\lambda_{m\ell}\right)}{\ell n\left(S_{k\ell} S_{m\ell}\right)} \qquad (15)$$

If this value is greater than n_k, the constrained maximum of $r(\Delta n)$ occurs at $\Delta n = n_{k\ell}$. This follows from

$$r'(0) = \lambda_{m\ell} - \lambda_{k\ell} > 0$$

and

$$r''(\Delta n) = \lambda_{m\ell} \left(\ell n\, S_{k\ell}\right) S_{m\ell}^{\;+\Delta n} + \lambda_{m\ell} \left(\ell n\, S_{m\ell}\right) S_{k\ell}^{\;-\Delta n} \leq 0$$

for all Δn in the range $0 \leq \Delta n \leq n_{k\ell}$ (since $0 \leq S_{ij} \leq 1$, $\ell n\, S_{ij} \leq 0$ and $\lambda_{ij} \geq 0$ for all i, j). Thus, $r(\Delta n)$ appears as in figure 2 or figure 3. Note that the curvature is always downward and that the maximum occurs at the point Δn^*. If the situation of figure 3 occurs, it is impossible to choose $\Delta n = \Delta n^*$ to force $\lambda_{k\ell}$ and $\lambda_{m\ell}$ to be equal. Consequently, equations (11) will not be satisfied. However, the optimal value of $f(n_{ij})$, where the n_{ij} are constrained by equation

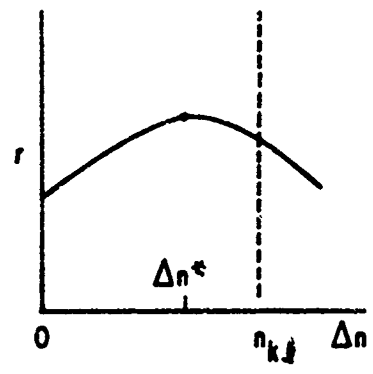

Figure 2. Constrained Maximum $\Delta n^* \leq n_{k\ell}$

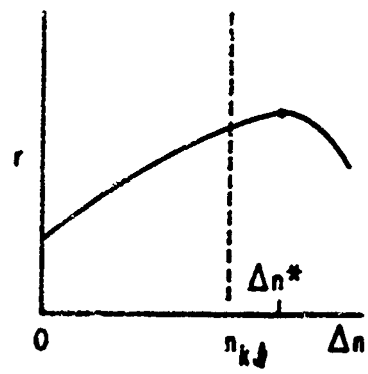

Figure 3. Constrained Maximum $\Delta n^* > n_{k\ell}$

(6) and equation (7), is still found due to the preceding comments concerning $r(\Delta n)$. Thus, the λ_{ij}'s merely serve as a means of adjusting the n_{ij}'s to approach optimality. The optimal n_{ij} are, of course, nonintegral and, therefore, not physically possible. Consequently, the optimal nonintegral allocation is integerized to give an integral allocation which satisfies the constraints. This integerization is performed for each weapon group j by rounding those n_{ij}'s with the largest fractional parts up and rounding the remaining n_{ij}'s down. Of course, the constraints

$$\sum_i n_{ij} = N_j$$

(in which the N_j's are integers) are satisfied by the real n_{ij}'s before integerization and must be satisfied by the integer n_{ij}'s also. Therefore, the number of n_{ij}'s rounded up is determined so that, for each weapon group j, the sum of those n_{ij}'s rounded up and those rounded down equals N_j. In practice, the expected kill resulting from this integerized allocation is not significantly different from the expected kill computed from the nonintegral allocation, since the difference in kills is usually only a fraction of an aircraft. Integerization of the optimal nonintegral allocation need not produce the optimal integral allocation, but it does produce at least a near-optimal integral allocation, with the difference in kills being the upper bound of how far from optimal the kill of the integerized allocation could be.

It has been indicated how $n_{k\ell}$ and $n_{m\ell}$ may be adjusted to increase the expected kill value when $\lambda_{k\ell} < \lambda_{m\ell}$ and $n_{k\ell} \geq 0.0001$ for some weapon group ℓ.

In practice, a tolerance level, ε, is set in QUANTO, so that convergence is said to occur when $\lambda_{m\ell} - \varepsilon \leq \lambda_{k\ell} \leq \lambda_{m\ell}$ for all $n_{k\ell} \geq 0.0001$ for all values of ℓ, where $\lambda_{m\ell} = \max \{\lambda_{i\ell}\}$, $(i = 1, 2, \ldots, M)$.

Specifically, the weapon group ℓ, upon which each allocation adjustment is based, is selected in a cyclical manner. The first allocation adjustment is made within weapon group one ($\ell = 1$) if $\lambda_{k\ell} < \lambda_{m\ell} - \varepsilon$ where

$$\lambda_{k\ell} = \min_i \left\{\lambda_{i\ell} \text{ such that } n_{i\ell} \geq 0.0001\right\}, \; i = 1, 2, \ldots, M \tag{16}$$

and

$$\lambda_{m\ell} = \max_i \left\{\lambda_{i\ell}\right\}, \; i = 1, 2, \ldots, M \tag{17}$$

If this situation does not exist for $\ell = 1$, successive weapon groups are inspected in sequential order until one is found in which the highest $\lambda_{i\ell}$ exceeds the lowest $\lambda_{i\ell}$ with a corresponding positive allocation ($n_{i\ell} \geq 0.0001$) by more than the tolerance ε. Successive allocation adjustments are accomplished in a repeating cycle through the values of ℓ (i.e., 1, 2, ..., L, 1, 2, ..., L, 1, 2, ...). Convergence occurs when all weapon groups are inspected without finding one which initiates an allocation adjustment. In practice, the λ_{ij} matrix if first converged to a tolerance of $\varepsilon = 0.1$, then $\varepsilon = 0.01$, then $\varepsilon = 0.001$ and so forth, with the final tolerance under the control of the user. This process results in a faster overall convergence to the final tolerance level. An additional cutoff of the convergence occurs if a given number (specified by the user, say 100) of allocation adjustments are performed without increasing the kill value above some user-selected amount (say 0.01). The user may also simply specify a maximum number of allocation adjustments to be made.

The iterative procedure is illustrated in the flow chart of figure 4, with several additional details appearing in the figure. When $\lambda_{k\ell} < \lambda_{m\ell} - \varepsilon$ is found for some weapon group ℓ, Δn must be computed. When only one type of aircraft is considered in the model, Δn is computed according to equation (12). However, when more than one type of aircraft is considered, Δn must be computed by a Newton iterative procedure, described in the mixed force allocation problem discussed in section VI.

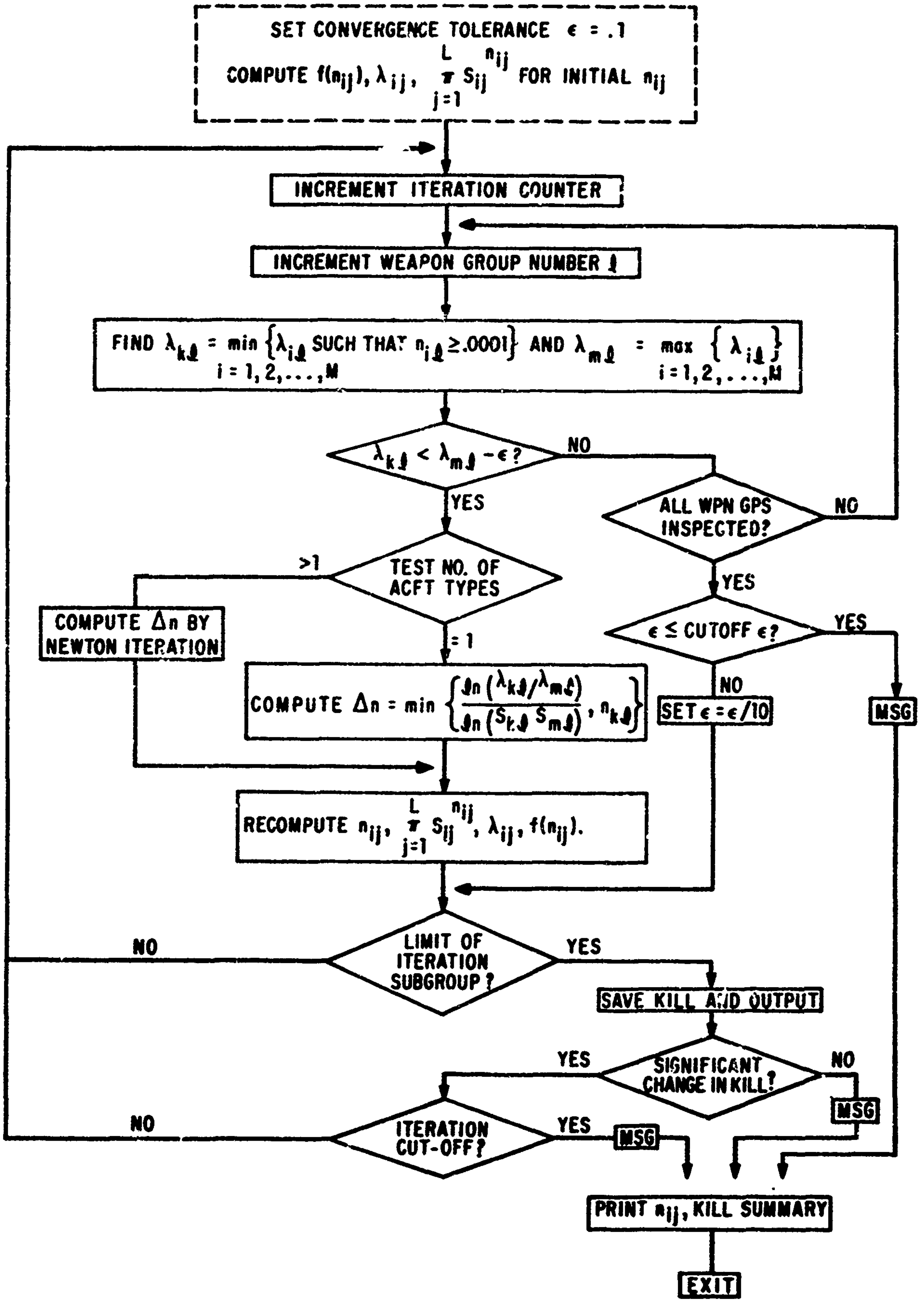

FIGURE 4. QUANTO's ITERATIVE PROCEDURE

The amount of output may be controlled to some extent by a control variable set by the user. Output on each iteration may be suppressed and only a limited output obtained after a "subgroup" of iterations. If the expected kill has not increased significantly for the subgroup of iterations, the procedure is terminated. Two other conditions may terminate the procedure, as shown in figure 4 and previously described.

3. OPTIMIZATION OF SUBMARINE LOCATIONS IN QUANTO

In the weapon allocation problem, the Lagrange multiplier λ_j represents the shadow value associated with weapon group j. In notation

$$\frac{\partial h}{\partial N_j} = -\lambda_j \geq 0 \tag{18}$$

where $h(n_{ij}, \lambda_j)$ is the Lagrangian function. Therefore, increasing N_j has the instantaneous effect of permitting an increase in h (and, therefore, f) at the rate of $-\lambda_j$ units of f per unit of N_j. Thus, one can get some feel for the value of an additional weapon in group j ob observing the magnitude of λ_j.

A heuristic rule has been used in QUANTO to relocate submarines among the input candidate submarine locations so as to improve the expected kill value. The value of a submarine at a given location bears some relation to the magnitudes of λ_{kj} for those weapon groups j corresponding to salvos from a submarine at that location. A submarine location input to QUANTO is characterized not only by its geographical coordinates but also by the type of submarine (and its number of salvos, s) which can be located there.

For each submarine, the quantity

$$\sum_j \lambda_{kj}$$

is calculated where the sum is over the s values of j corresponding to that submarine's salvos, and each

$$\lambda_{kj} = \min_i \left\{\lambda_{ij} \text{ such that } n_{ij} \geq 0.0001\right\},\ i = 1, 2, \ldots, M$$

for each weapon group j at that submarine's current location. Similarly, for every other location at which that submarine can operate, the quantity

$$\sum_j \lambda_{mj}$$

is calculated, where

$$\lambda_{mj} = \max_i \left\{\lambda_{ij}\right\}, \ i = 1, 2, \ldots, M$$

with a view to possibly moving the submarine to a location where it can be expected to kill more value. When there are several types of submarines, the type of submarine moved is the one having the largest average difference

$$\frac{1}{s}\left\{\sum_j \lambda_{mj} - \sum_j \lambda_{kj}\right\}, \ \lambda_{kj} \text{ such that } n_{kj} \geq 0.0001$$

Within this submarine type, the submarine relocated is the one corresponding to the lowest quantity

$$\sum_j \lambda_{kj}$$

and it is placed in the location having the highest value of

$$\sum_j \lambda_{mj}$$

Relocation of a submarine in QUANTO is accomplished by moving the s missiles on that submarine to another location. Consequently, n_{ij} is increased by one missile in the s weapon groups j corresponding to the submarine location to which that submarine is moved and those additional SLBMs are assigned to targets i having the largest value λ_{ij} (for each j). Similarly, for the s weapon groups j from which a missile is removed, n_{ij} is reduced for the targets i corresponding to the lowest λ_{ij}'s until a total of one missile is removed from each weapon group. In this way, a rational guess is made at where the missiles from the relocated submarine should go in order to obtain an initial allocation prior to re-entering the laydown optimization procedure.

This submarine relocation process in no way guarantees an increase in value killed. This is because the relocation is accomplished by moving an integral number of missiles, not a Δn computed to maximize kill. Also, s missiles (not just one) are moved before the λ_{ij} are recomputed from the new n_{ij}'s.

Although the heuristic rule does not always increase the kill, experience with the procedure reveals that the kill usually increases with every more until a decrease occurs, after which the kill varies with additional moves without significant gains or losses. Consequently, submarine moving is terminated in QUANTO after the first move which results in a decreased kill.

4. BEDDOWN OPTIMIZATION

A heuristic routine for shifting aircraft from base to base has been supplied in the QUANTO weapon allocation code with the intent of determining better aircraft beddowns for a given positioning of submarines. The procedure shifts aircraft from the base having aircraft value greater than 0.0001 with the lowest survivability product

$$\prod_{j=1}^{L} S_{ij}^{\,n_{ij}}$$

to the base with the highest survivability product, provided the losing base starts with an aircraft value greater than 0.0001. Otherwise, the survivability products are inspected in ascending order until a corresponding aircraft value greater than 0.0001 is found, and the corresponding base is selected as the losing base. The amount of value shifted is the nonintegral product

$$\Delta V = V_k \cdot \left(\prod_{j=1}^{L} S_{mj}^{\,n_{mj}} - \prod_{j=1}^{L} S_{kj}^{\,n_{kj}} \right) \tag{19}$$

where bases m and k are those having the highest and lowest survivability products, respectively, where $V_k > 0.0001$. If $V_k < 0.1$, then all the value V_k is moved from base k to base m regardless of the ΔV computed.

A with the heuristic submarine relocation routine, each shift of beddown value in accordance with the above formula does not guarantee a decrease in the overall expected kill value, although the general trend is toward a lowering of the kill. Occasionally, an overall kill increase may occur as the result of individual shifts considerably before the process has exhausted the gains to be made in aircraft surviving. The shifting of value terminates if the value ΔV to be moved is less than 0.05 (specified by a program statement), at which point the survivability products have essentially converged and the beddown is not changing significantly.

Shifts of aircraft do not cause a recomputation of the survivabilities S_{ij} in QUANTO, since only rarely does the computed survivability depend upon the number of aircraft present at the target. The methods of computing the survivabilities S_{ij} are described in the next section of this report.

If both beddown optimization and optimization of submarine locations are requested by the user of QUANTO, the beddown optimization is performed last. Of course, if the user wants the submarines to have the last move, he may request beddown optimization only, and in a subsequent run, input the optimal beddown and request submarine optimization.

The beddown optimization procedure shifts nonintegral numbers of aircraft, and thus results in a beddown which has fractional numbers of aircraft at the various bases. After the termination of aircraft moves, the beddown is integerized along with the missile laydown and the results are output. Integerization of the beddown has the effect of increasing the kill by a negligible amount over the expected kill computed on the basis of nonintegral beddown.

The following discussion, in the form of a critiqued proof, is presented as a partial justification for the heuristic rule for improving the beddown. Suppose $(V_1, V_2, \ldots, V_k, \ldots, V_m, \ldots, V_M)$ represents the values of the aircraft bedded down on the M bases for which the optimal missile laydown is $[n_{ij}]$ and

$$\prod_{j=1}^{L} S_{kj}^{\;n_{kj}} < \prod_{j=1}^{L} S_{mj}^{\;n_{mj}} \tag{20}$$

Next suppose the beddown is changed by subtracting some small value $\varepsilon > 0$, from V_k and adding ε to V_m. Thus, $(V_1, V_2, \ldots, V_k - \varepsilon, \ldots, V_m + \varepsilon, \ldots, V_M)$ represents the new beddown, and the new optimal laydown $[\hat{n}_{ij}]$ could be determined. The new value surviving in the new beddown is then

$$\text{New Surviving Value} = \sum_{i=1}^{M} V_i \prod_{j=1}^{L} S_{ij}^{\;\hat{n}_{ij}} + \varepsilon \prod_{j=1}^{L} S_{mj}^{\;\hat{n}_{mj}} - \prod_{j=1}^{L} S_{kj}^{\;\hat{n}_{kj}} \tag{21}$$

Now if ε is sufficiently small, it is reasonable to expect that $[\hat{n}_{ij}]$ is close to $[n_{ij}]$, so that

$$\prod_{j=1}^{L} S_{mj}^{\hat{n}_{mj}} - \prod_{j=1}^{L} S_{kj}^{\hat{n}_{kj}} \cong \prod_{j=1}^{L} S_{mj}^{n_{mj}} - \prod_{j=1}^{L} S_{kj}^{n_{kj}} > 0$$

Therefore, for some choice of ε

$$\text{New Surviving Value} > \sum_{i=1}^{M} V_i \prod_{j=1}^{L} S_{ij}^{\hat{n}_{ij}} \tag{22}$$

Furthermore,

$$\sum_{i=1}^{M} V_i \prod_{j=1}^{L} S_{ij}^{\hat{n}_{ij}} > \sum_{i=1}^{M} V_i \prod_{j=1}^{L} S_{ij}^{n_{ij}} = \text{Old Surviving Value} \tag{23}$$

since $[n_{ij}]$ is the optimal laydown for the old beddown (V_i) and therefore minimizes the survivors. Consequently, a shift of value (sufficiently small) from base k to base m, when

$$\prod_{j=1}^{L} S_{mj}^{n_{mj}} > \prod_{j=1}^{L} S_{kj}^{n_{kj}}$$

results in a reduced expected kill. Note that this proof does not indicate the best amount of value ε to shift, but merely that value should be shifted to bases having high survivability products from those with low products.

5. LETHAL AREA DETERMINATION

The determination of lethal area (i.e., the region within which aircraft are destroyed) resulting from a nuclear weapon detonation is an integral part of the QUANTO code. The lethal areas are required in the computations of the survivabilities, S_{ij}, of aircraft flushing each target area. This subsection discusses the assumptions, assertions, models, and methods used in the lethal area determination.

The nuclear environment created by the detonation of nuclear weapons is discussed in AFSCM 500-1. This section is concerned with the fireball effect (thermal) and the blast (overpressure) effect which are considered to be the only two structural kill mechanisms which can destroy an aircraft for this model.

A superheated region, the fireball, cools while expanding and radiates thermal energy (heat). At the expanding edge of the fireball, tremendous pressures are created and form a shock front. The shock front propagates approximately spherically at supersonic speeds and produces a crushing overpressure force with accompanying gusts of dynamic forces. The thermal energy effect is measured in calories per square centimeter (cal/cm^2), and blast effects in pounds per square inch (psi).

Mathematical models are available to study the thermal and blast effects. Computer codes for these models require many hours of computer time; consequently, the precise codes are not suited for systems analysis or war games. Reliable models based on the precise hydrodynamic and radiation hydrodynamic models have been developed, tested, and improved by the Air Force Weapons Laboratory. Codes for these models require only milliseconds of computer time and, therefore, are suitable for systems analysis. The computerized versions of these codes bear the names SABER and SNAPT, and are used to approximate the blast and thermal environments, respectively.

The Systems Analysis Blast Environment Routine (SABER) has been modified to determine only the ranges of given levels of overpressure and the times of shock arrival at those ranges. This is a restricted use of the multipurpose program. The modified version is called SABERCM. Inputs, in addition to specified peak overpressure, are nuclear weapon yield, height of burst, terrain height, and aircraft altitude. Outputs are overpressure range and time of shock arrival.

SNAPT is a computerized model which can be used to calculate the free-field thermal energy resulting from the detonation of a nuclear weapon or to calculate the range at which a given level of free-field thermal energy occurs. SNAPT has been modified to perform only the latter calculation as a subroutine named SNAPTCM. Necessary input data, other than the free-field energy level, consist of nuclear weapon yield, height of burst, terrain height, aircraft altitude, and pertinent atmospheric conditions. The atmospheric parameters include haze layer height, water vapor pressure, ground reflectance (albedo), and visibility. The horizontal range at which the desired free-field thermal energy level occurs is output from SNAPTCM.

The cookie-cutter assumption has been made to distinguish between regions of lethality and nonlethality to aircraft. For these purposes, aircraft vulnerability levels for the thermal and overpressure kill mechanisms are specified in cal/cm^2 and psi, respectively. Under the cookie-cutter assumption, aircraft are assumed to be killed if either specified vulnerability level is exceeded, and safe otherwise.

Vulnerability levels are input to the nuclear routines along with atmospheric conditions, terrain height, and weapon characteristics in order to calculate the lethal nuclear environmental ranges for a fixed height of receiver. A point of detonation is first specified for each missile at each potential target, based upon where the aircraft from that airfield are located when the missile arrives. The routines SABERCM and SNAPTCM require a height of receiver to compute the horizontal ranges of the lethal nuclear environment. The height of receiver is taken as the altitude of the aircraft, according to its flight profile, at the time of weapon detonation (relative to the brake release time). This is equivalent to slicing the spherically propagating shock front (or overpressure contour) and the thermal contour with a plane parallel to the ground at a distance above the ground equal to the altitude of the aircraft at the above. This horizontal plane, called the lethal plane, is the geometrical structure in which the determination of lethal area is accomplished.

The general appearance in the lethal area plane of the overpressure and thermal contours, at the lethal levels specified by the vulnerability levels and relative to the other input data, is that of two concentric circles centered at the perpendicular point projection of the burst center onto the plane. Figure 5 depicts the intersection of the lethal area plane with the lethal overpressure contour. The horizontal range associated with the lethal overpressure contour is the lethal overpressure radius, and is measured from the perpendicular point projection of the burst center onto the plane to the lethal overpressure contour. Similarly, the lethal thermal radius is that horizontal range associated with the lethal thermal contour. Thus, the nuclear routines are used to compute the lethal contours needed to compute lethal area.

The orientation of the aircraft is not considered in computing the horizontal ranges associated with the overpressure or thermal kill mechanisms. The lethal thermal radius is computed under the assumption that the aircraft is oriented

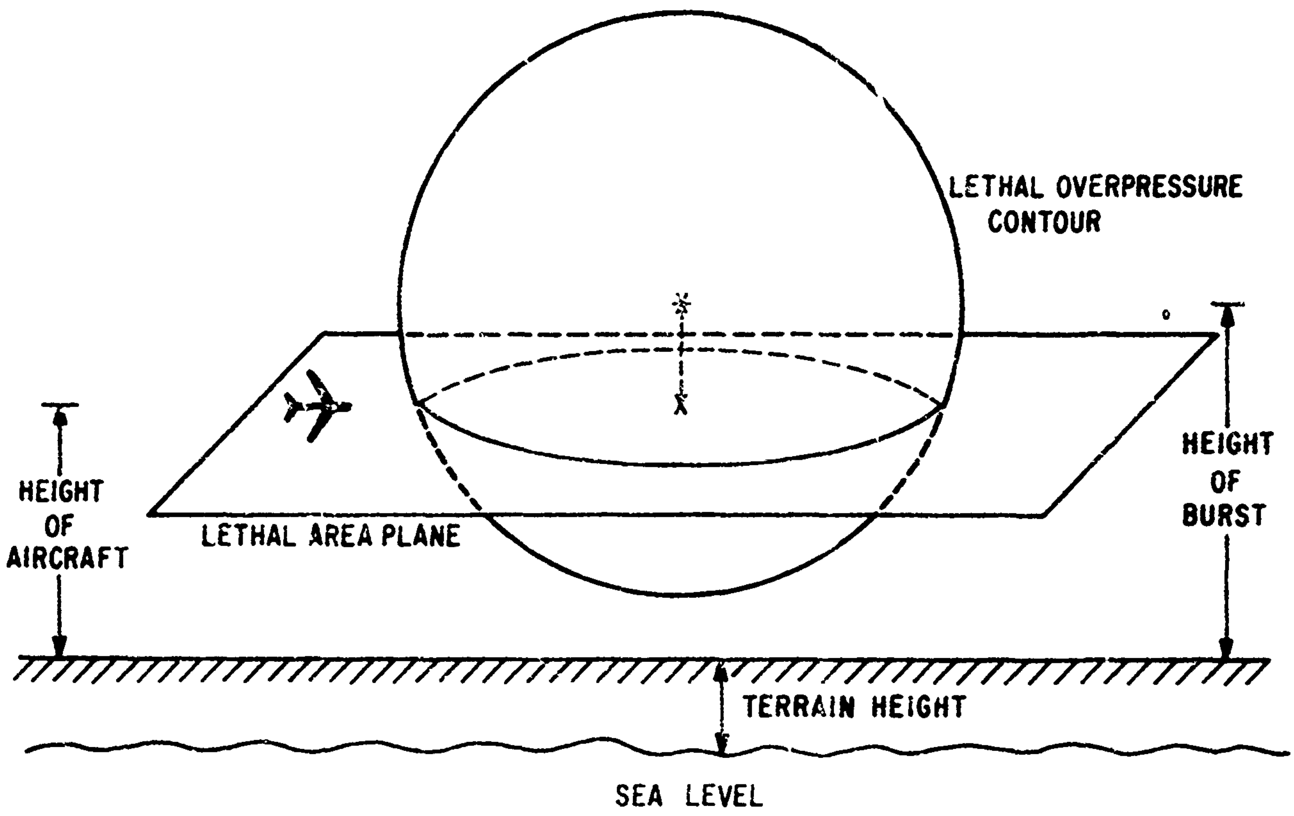

Figure 5. Lethal Area Plane Intersecting Overpressure Contour

so as to receive the maximum amount of thermal energy. Similarly, the fact that the aircraft is better equipped to withstand the overpressure shock front in one position, as opposed to another, is also not taken into account in the computation of the lethal overpressure radius.

The lethal radii which are outputs from the nuclear routines are computed for a stationary receiver (aircraft). Thus, the lethal contours defined by the lethal radii must be adjusted to account for a moving aircraft. This is accomplished in the lethal area plane.

The letha [illegible]a (within the lethal area plane) resulting from the detonation of a nuc[illegible] weapon is that area within which the aircraft cannot survive if located there at the onset of the detonation. Tabulated data for the aircraft flight profile is used in conjunction with the distance to the centroid (defined in section V, fifth assumption) to transform the lethal overpressure contour in the lethal area plane into a locus of points describing the boundary of the area reflecting aircraft kill from overpressure. The lethal thermal

contour remains unchanged since the thermal energy propagation time is negligible. The entire process of lethal area determination takes place in the lethal area plane.

A top view of a typical lethal area plane containing a lethal overpressure contour, a lethal thermal contour, and a centroid (designated by an asterisk) is offered in figure 6. The lethal overpressure contour is represented by the dashed-line circle, the lethal thermal contour by the solid-line circle, and the perpendicular point projection of the burst center onto the lethal area plane by the symbol X. The remaining task is to adjust the overpressure contour to account for the movement of the aircraft.

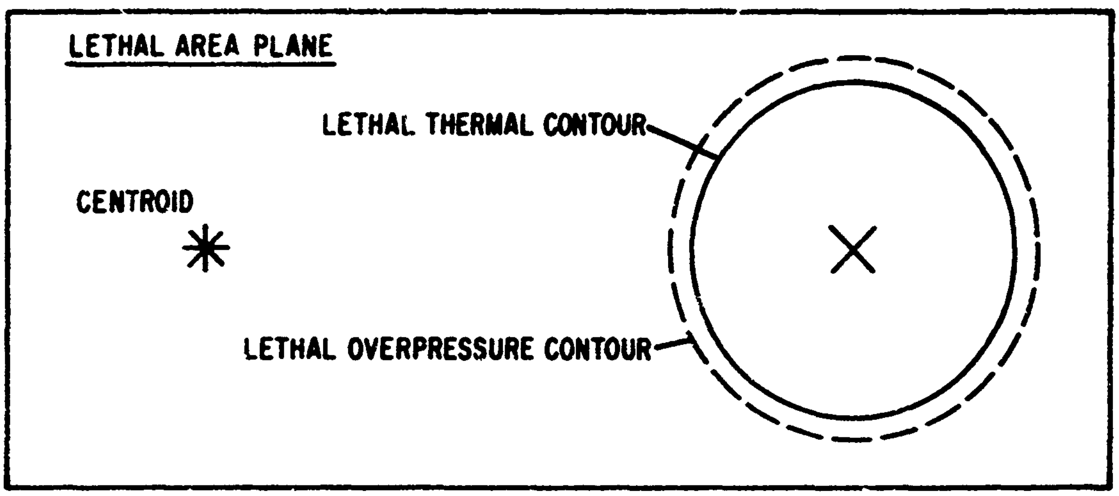

Figure 6. Top View of Lethal Area Plane

Points lying on the lethal overpressure contour are used to determine the boundary for the overpressure lethal locus, that is, the lethal area at the onset of the burst associated with the overpressure kill mechanism. The aircraft are assumed to be emanating radially from a point called the centroid. The points on the lethal overpressure contour are backed up radially toward the centroid by the distance flown between detonation and the arrival of the shock wave at the overpressure contour. This distance is obtained by interpolation from the distance/time coordinates representing the aircraft flight profile. This radial translation of the overpressure contour toward the

centroid usually results in a petal or egg-shaped overpressure lethal locus as shown in figure 7. More complex shapes may result when the centroid is within the lethal overpressure contour.

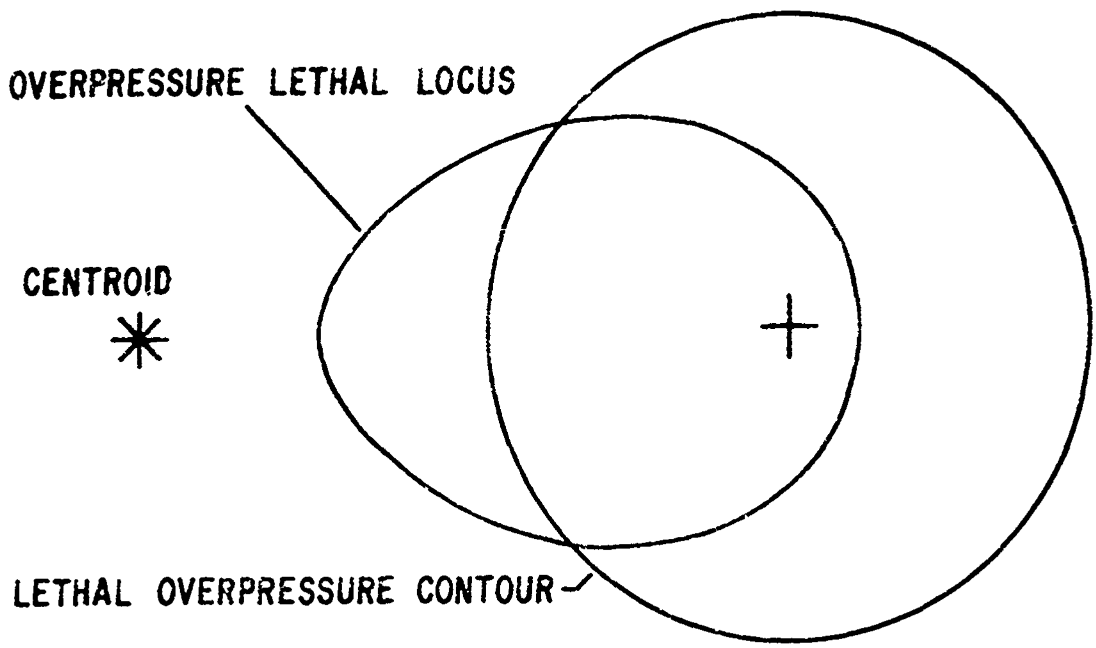

Figure 7. Overpressure Lethal Locus

The overpressure lethal locus encompasses the overpressure kill region at the time of detonation under the cookie-cutter assumption for a moving aircraft. An underlying assumption is that the aircraft maintains radial flight from the centroid. Since the aircraft is constrained by the aircraft flight profile, if within the overpressure lethal locus at the onset of the burst, it will be intercepted by the supersonically propagating shock front at a higher level of overpressure than it can withstand. A possibility exists that the aircraft could be located within the lethal thermal contour, as well as within the overpressure lethal locus, at the onset of the detonation.

The overpressure lethal locus is combined with the lethal thermal contour to produce the boundary of the lethal area. Figure 8 gives an example of a lethal circle/petal area with respect to a moving aircraft. Numerical integration is used to compute the area within this lethal region.

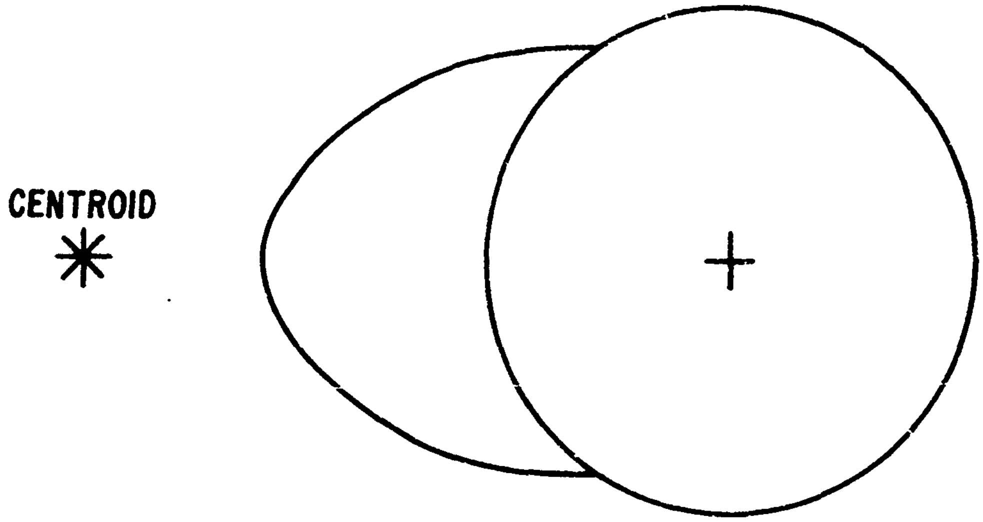

Figure 8. Lethal Area for Moving Aircraft

6. DETERMINATION OF SURVIVABILITIES

The values S_{ij}, which appear in the objective function (maximizing the expected value killed), must be computed for each weapon in group j against each target i. Assuming that the aircraft at each target are uniformly distributed over some area at each weapon arrival time, the probability of kill, P_k, of each aircraft at that target is the quotient of the lethal area divided by the area in which the aircraft could be located when the weapon arrives. In this report, the P_k will be defined as the probability of kill given that the weapon successfully detonated at the target. In QUANTO, reliability factors are given for each missile type for the probabilities that (1) the missile is successfully launched, (2) the missile successfully reaches the target, and (3) the warhead successfully detonates. The overall reliability of a missile is the product of these three reliabilities. The survivability of a target from a single weapon is then

$$\text{Survivability} = 1 - P_k * (\text{reliability})$$

The P_k of each weapon versus each target is computed from input data in QUANTO. Initially, the arrival time of the missile on the target is computed. Time zero is the time at which all of the first missiles from each submarine are simultaneously launched. Subsequent salvos from the submarines are launched after time zero, as determined by the salvo number and the missile launch

interval. The flight time of a missile to a given target is interpolated from input distance/time missile trajectory information after the distance from launch point to target (i.e., the coordinates of the base) is computed. The arrival time of the weapon on the target is the sum of the launch time and the flight time.

The location of the aircraft at the time of weapon arrival may be computed from the input aircraft flight profile and the brake release times. The aircraft are assumed to disperse radially from a single point, called the centroid. Unless aircraft are assumed to be departing in both directions from a base (from dual runways, for instance), the centroid will not be on the runway, for the centroid's location is a function of the time it takes an aircraft to raise its gear and flaps, reach a turn altitude, etc., and then make a turn to its fly-out direction. The distance an aircraft will be from the centroid at weapon arrival time is computed by (1) subtracting the brake release time from the weapon arrival time to obtain the time the aircraft has had to escape before the weapon arrives, (2) interpolating in the aircraft flight profile to obtain the distance the aircraft has traveled from brake release, and (3) subtracting the distance from brake release to centroid from the total distance traveled. The distance to the centroid from brake release point is input for each target. In this manner, QUANTO computes:

R_1 = the radial distance from the centroid of the first aircraft at weapon arrival time

and

R_N = the radial distance from the centroid of the last aircraft at weapon arrival time

If an aircraft has either not begun its takeoff or not reached the centroid, its radial distance from the centroid is set to zero. The intent here is to treat all aircraft which have not reached the centroid as essentially undispersed aircraft which can be targeted with a single SLBM.

The area of kill generated by a warhead detonation for aircraft of a given type is dependent on many parameters, as described in the preceding section of this report. Many of these parameters are needed to describe the nuclear environment and are directly supplied by user inputs. First, the horizontal

ranges are determined at which a stationary receiver would experience a lethal overpressure or thermal effect. Then the assumed detonation point of the SLBM, the aircraft climb profile, and the distance from brake release point to the centroid are used as described in the previous section to determine the shape of the circle/petal thermal/overpressure lethal area at detonation time, thus taking into account the moving receiver (aircraft). The lethal area varies somewhat with the distance of the detonation from the centroid, because the aircraft are at different altitudes and velocities at different points in the climb profile. Hence, an approximation must be made of the lethal area used in the calculation of P_k, and QUANTO must make some assumption about where the weapon might land without having determined yet how many total SLBMs will be allocated to the target.

QUANTO assumes that the attacker can compute R_1, assuming a certain brake release time, and accepts this as the farthest distance that the first aircraft on a base can achieve by weapon arrival time. However, although the attacker might also be able to compute R_N based on the stipulated aircraft takeoff intervals and brake release time, he would realize that unanticipated delays could occur (in detection and warning of attack, etc.) and might consider it equally likely to find aircraft at any point within the circle of radius R_1. Consequently, QUANTO attacks that area, in general, with a uniformly dense distribution of weapons. Thus, an average weapon might land at a distance $(R_1/\sqrt{2})$ from the centroid because a circle of radius $(R_1/\sqrt{2})$ contains half the area within the circle of radius R_1. One benefit of this uniform attack is that a delay in the brake release time will not significantly decrease the expected kill and may result in a large increase in kill. When the SLBM detonates at distance $(R_1/\sqrt{2})$ from the centroid and the entire lethal circle/petal area falls within the circle of radius R_1, the P_k is simply

$$P_k = \frac{A_L \left[\text{for } R_1/\sqrt{2}\right]}{\pi R_1^{\,2}}$$

where

$$A_L \left[\text{for } R_1/\sqrt{2}\right] \tag{24}$$

indicates the lethal area when the detonation point is at range $(R_1/\sqrt{2})$ from the centroid. This situation is illustrated in figure 9.

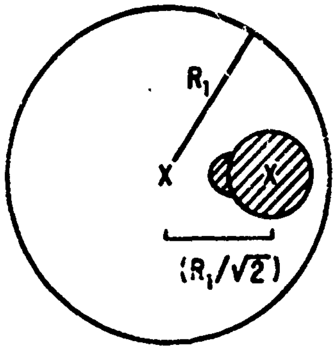

Figure 9. Detonation Point at $R_1/\sqrt{2}$

The use of equation (24) results in an overestimate of the expected kill of aircraft in the case of very few aircraft on a base and a lethal area which is a large portion of the circular area πR_1^2. With few (or one) aircraft, R_N may be only slightly less than (or equal to) R_1. This situation is shown in figure 10 for the lethal area labeled A. In this case, equation (24) predicts a large percentage of the aircraft killed, although when the brake release time is certain, no kills result. To guard against this possibility, the attacker would wish to reduce his estimate of P_k in allocating his attacking weapons. One way in which he could do this would be to replace the procedure of the previous paragraph with one in which the weapon was placed at distance $(R_1 + R_N)/2$ from the centroid, assuming the aircraft were uniformly distributed throughout the annulus of radii R_1 and R_N. This P_k is the shaded area within the circle/petal labeled B in figure 10, divided by the annulus area $\pi\left(R_1^2 - R_N^2\right)$. The shaded area within B may be approximated by considering the circle/petal B as a circle of equivalent area centered at $(R_1 + R_N)/2$ radial distance from the centroid, and computing the area within both the equivalent circle and the annulus thickness. This latter common area, which will be labeled $A_{LAN}\left[\text{for } (R_1 + R_N)/2\right]$, may be computed from closed-form geometric expressions. The single aircraft situation is handled in this manner by artificially setting $R_N = R_1 - 0.01$ so that the annulus has a small positive area. The P_k formula thus becomes

$$P_k = \min\left\{\frac{A_L\left[\text{for } R_1/\sqrt{2}\right]}{\pi R_1^2}, \frac{A_{LAN}\left[\text{for } (R_1 + R_N)/2\right]}{\pi R_1^2 - R_N^2}\right\} \tag{24}$$

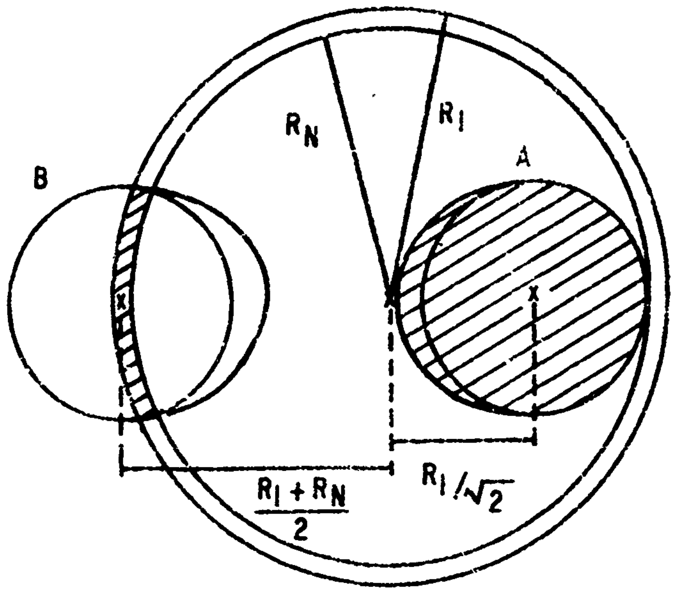

Figure 10. Lethal Area Detonation Illustration

The use of this formula, when the second quotient is the minimum of the two, encourages OUANTO to allocate a second weapon to the target since the first weapon's P_k is lowered.*

As shown in the previous figures, the thermal lethal circle usually extends farther from the centroid than does the overpressure lethal petal. Likewise, when the SLBM detonates at the centroid, the thermal circle usually extends farther from the centroid than does the region of overpressure kill, which in this special case is also a circle. "Usually" in this context means for those combinations of flight profile and overpressure/thermal vulnerability levels normally of interest. However, for some hardness levels and aircraft flight profiles, the region of overpressure kill may totally encompass the thermal lethal circle.

The farthest reach of the lethal region in a direction away from the centroid becomes a concern when R_1 is small enough that the lethal area may protrude beyond R_i for a given weapon placement. When the lethal area so protrudes, the

*The validity of the method implicit in the second term, and indeed of the whole procedure in equation (24), has been confirmed by comparing answers obtained from QUANTO with those obtained using a Monte Carlo simulation model.

first expression competing for the minimum in equation (24) is in error because the aircraft cannot be located in the protruding portion of the lethal area. To handle these cases, the weapon is assumed to detonate at a position for which no protrusion occurs, if possible.

Therefore, QUANTO computes

R_{LR} [for $R_1/\sqrt{2}$] = the distance from detonation point to lethal region boundary, in a direction away from the centroid, for a detonation at distance $(R_1/\sqrt{2})$ from the centroid.

It will usually be true that

$$R_{LR}\left[\text{for } R_1 - R_{LR}\left[\text{for } R_1/\sqrt{2}\right]\right] \simeq R_{LR}\left[\text{for } R_1/\sqrt{2}\right]$$

i.e., R_{LR} varies little as the detonation point is adjusted to avoid protrusion. Now, if

$$R_1 \leq R_{LR}\ [\text{for } 0],\ \text{then QUANTO sets } P_k = 1$$

But if

$$R_1 > R_{LR}\ [\text{for } 0]$$

and

$$(R_1/\sqrt{2}) + R_{LR}\left[\text{for } R_1/\sqrt{2}\right] > R_1 \text{ (i.e., the lethal area protrudes beyond } R_1)$$

then QUANTO computes the P_k as

$$P_k = \min\left\{\frac{A_L\left[\text{for } R_1 - R_{LR}\left[\text{for } R_1/\sqrt{2}\right]\right]}{\pi R_1^2}, \frac{A_{LAN}\left[\text{for } (R_1 + R_N)/2\right]}{\pi\left(R_1^2 - R_N^2\right)}\right\} \tag{25}$$

where the first quotient assumes placement of the weapon at

$$\left(R_1 - R_{LR}\left[\text{for } R_1/\sqrt{2}\right]\right)$$

in order to avoid protrusion.

Thus, the complete formula for P_k is

$$P_k = \begin{cases} 1 & \text{if } R_1 \leq R_{LR}\,[\text{for } 0] \\ \min\left\{\dfrac{A_L\left[\text{for } R_1 - R_{LR}\,[\text{for } R_1/\sqrt{2}]\right]}{\pi R_1^2}, \dfrac{A_{LAN}\left[\text{for } (R_1 + R_N)/2\right]}{\pi\left(R_1^2 - R_N^2\right)}\right\} & \\ \qquad \text{if } R_1 > R_{LR}\,[\text{for } 0] \text{ and } R_1/\sqrt{2} + R_{LR}\,[\text{for } R_1/\sqrt{2}] > R_1 & \\ \min\left\{\dfrac{A_L\,[\text{for } R_1/\sqrt{2}]}{\pi R_1^2}, \dfrac{A_{LAN}\left[\text{for } (R_1 + R_N)/2\right]}{\pi\left(R_1^2 - R_N^2\right)}\right\} & \text{otherwise} \end{cases} \tag{26}$$

It may be noted in the above formula that the weapon placement may be at distances from the centroid of 0 (when $P_k = 1$), $R_1 - R_{LR}\,[\text{for } R_1/\sqrt{2}]$, $(R_1 + R_N)/2$, or $R_1/\sqrt{2}$. This will be of concern when more than one aircraft type is considered in the model.

It should be noted that these formulas for P_k are inaccurate when the distribution of aircraft is far from uniform over an area, as might be the case if a number of aircraft had not left the base by the time of a weapon arrival. This situation will not occur if the aircraft beddown is a rational one, intended to prevent mass kills by single weapons. Because of the assumption of uniform distribution of aircraft, QUANTO will underestimate the aircraft kills in these situations. However, a simulation program may be used to discover if such conditions exist and to estimate the resultant kills.

If the aircraft were actually uniformly distributed over the areas assumed, the actual P_k values (and, hence, S_{ij} values) realized by the allocated weapons would not agree exactly with those computed by formulas (23) and (24). This is due to the impossibility of determining the realized lethal area sizes before determining the number of weapons (and, therefore, the precise placement of weapons) on each target. The computed weapon allocation is optimal for the P_k values computed. However, the plus and minus errors between realized and

computed P_k's for individual weapons tend to balance out to a small overall error when summed over all the weapons. This is because the computations of P_k's are based on an average placement of each weapon on each target.

It has been assumed that the survivabilities S_{ij} are independent. Thus, no weapon on target i can cause collateral damage on aircraft from another base, and the area purged of aircraft by a detonation can become populated to an equal aircraft density by other aircraft before the next weapon arrives. Detailed base-by-base simulation of the attacks produced by QUANTO has shown that the actual resultant kill (output from the simulator) is not significantly different from QUANTO's predicted kill.

SECTION IV

THE COMPUTER PROGRAM

Figure 11 shows, in general terms, the operation of QUANTO. After the data for the problem is input, areas of lethality and the survivabilities S_{ij} are computed. The optimal missile laydown n_{ij} is then determined. QUANTO will then relocate a submarine to a better position, if the user has requested submarine optimization, and recompute the optimal n_{ij} for the new positions of the submarines. After submarine optimization is completed, aircraft may be relocated to improve the number surviving, with n_{ij} recomputed following each shift of aircraft. When beddown has been optimized, the optimal nonintegral V_i and n_{ij} are integerized and final output is produced.

Figure 12 indicates several additional details of QUANTO. The criteria for terminating submarine optimization and beddown optimization are indicated in test blocks. A mode parameter, input on the first data card of a problem deck, controls from whence input is taken and how much of the program is executed. Table II described the mode options. These options permit the user to observe partial computations for validity without risking a large expenditure of computer time.

The principal subroutines of QUANTO (QUANTO being the name of the main program) and their functions are listed in table III.

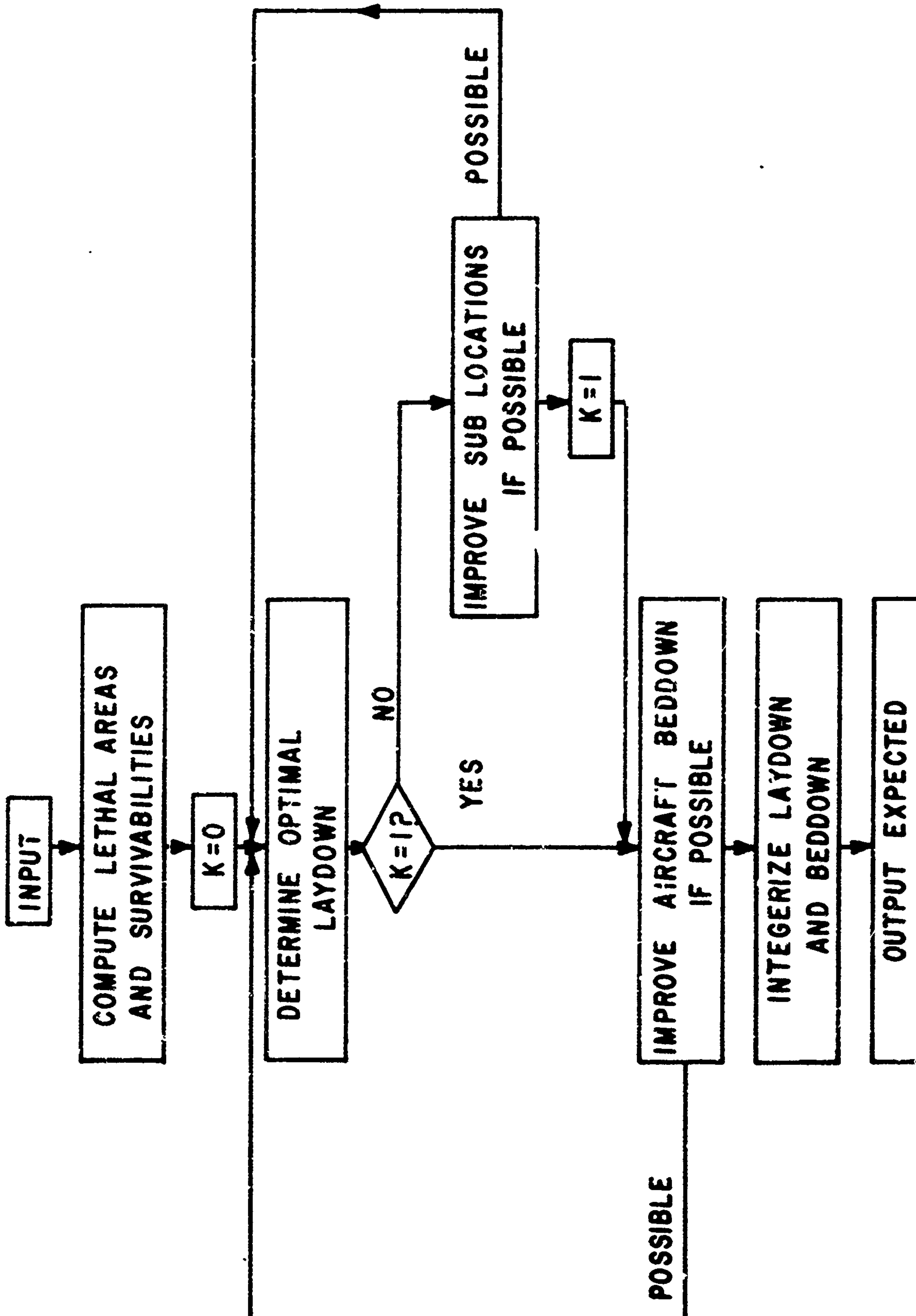

Figure 11. QUANTO Flow

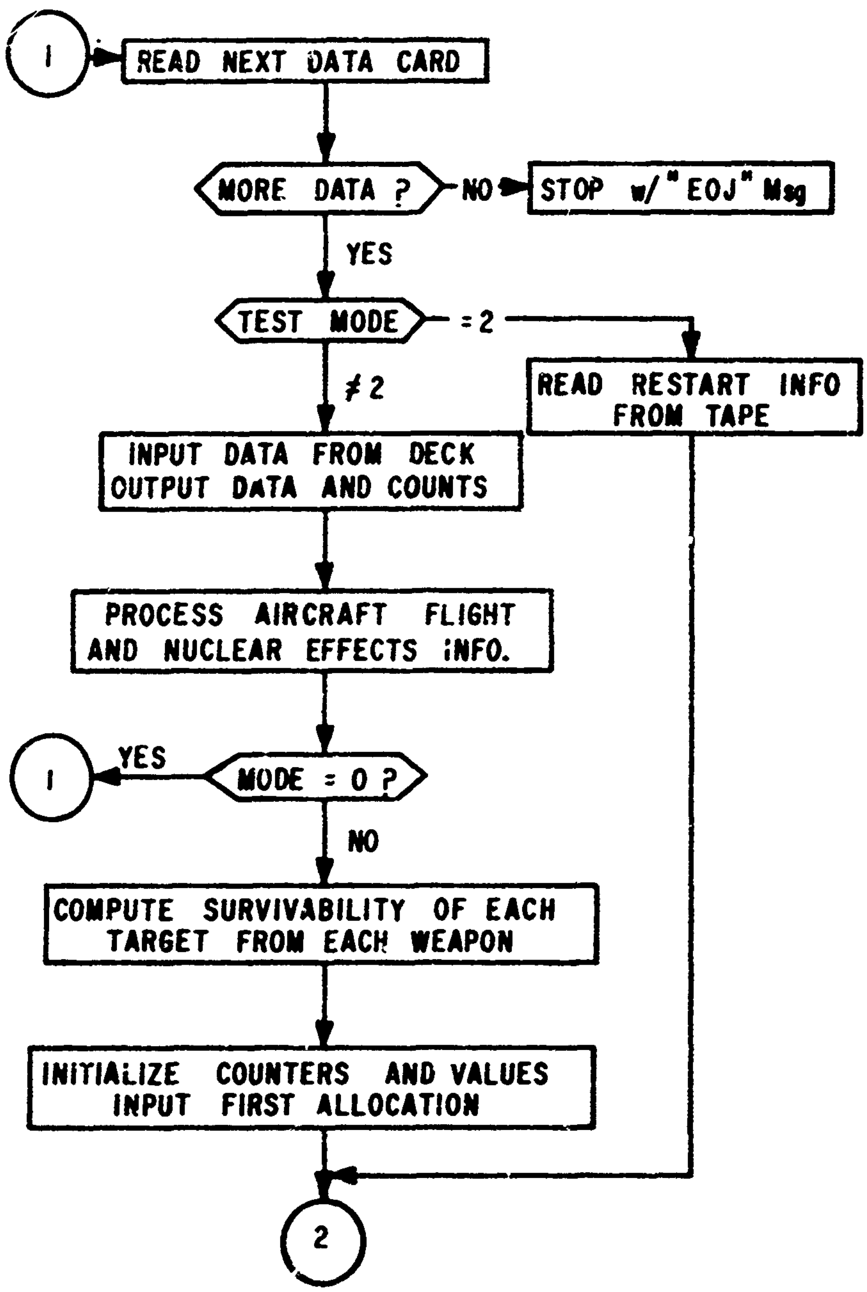

Figure 12. Detailed QUANTO Flow Chart

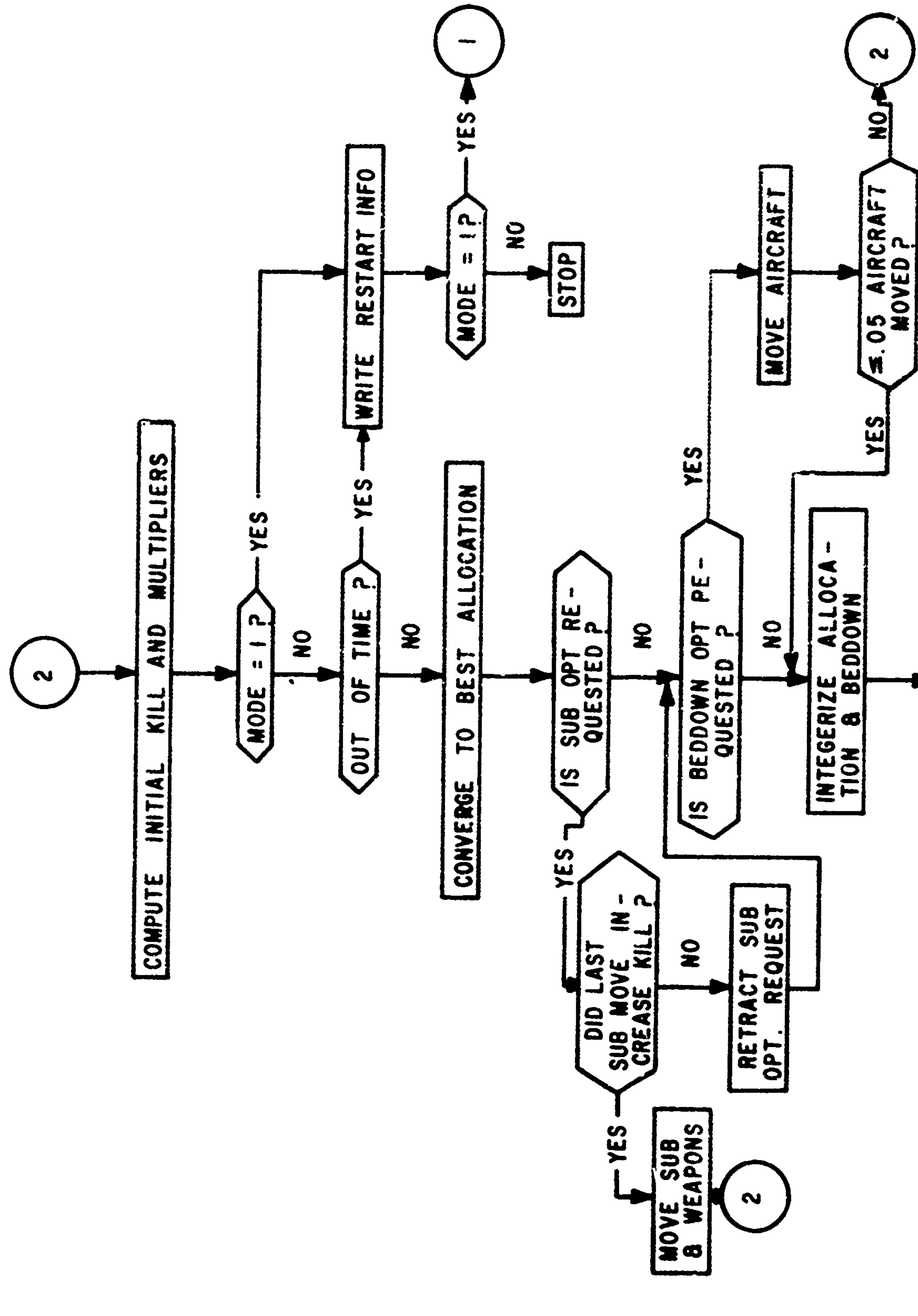

Figure 12. Detailed QUANTO Flow Chart (cont'd)

Table II

MODE OPTIONS

Mode	QUANTO Operations
0	Input all data except convergence parameters and initial allocation n_{ij} from cards.
	Terminate problem after computing nuclear effects and constructing flight profiles.
1	Input all data from cards.
	Terminate after computing survivabilities S_{ij} and kills resulting from initial allocation.
	Write information for automatic program restart on tape.
2	Input all data (except parameters on first card) from restart tape.
	Terminate problem computations prior to completion only if the time limit for processing is reached, at which time a restart tape will be written.
3	Input all data from cards.
	Terminate problem as for mode = 2.

Table III

PRINCIPAL ROUTINES AND THEIR FUNCTIONS

Routine Name	Description
QUANTO	Main Program.

QUANTO reads the input data, either from cards or from a restart tape, as controlled by the mode parameter on the first input card of each problem deck. The data describing the problem are printed, and if the data are read from cards, a summary of the input data is also printed. The aircraft profile and parameters affecting nuclear effects are not read by QUANTO, but by PROCESS, called by QUANTO.

Computations of survivabilities S_{ij} are performed mainly in QUANTO. DETAREA provides QUANTO the necessary lethal areas, but QUANTO computes flight times and distances and the resultant set of P_k and S_{ij} values, with the help of interpolation, look-up, and distance computational routines.

After input of the initial allocation, QUANTO controls the sequencing of operations in the iterative procedure for optimizing the missile laydown. When provided Δn by ADJLAM, QUANTO recomputes n_{ij}, S_{ij}, λ_{ij}, and $f(n_{ij})$. Control of the iteration cutoff and intermediate output is accomplished in QUANTO.

Relocation of aircraft is completely performed in QUANTO, but QUANTO calls other routines for submarine optimization and integerization of laydown and beddown.

Table III (cont'd)

Routine Name	Description
PROCESS	Called by QUANTO. PROCESS reads aircraft profile data and nuclear effects parameters from cards for each type of aircraft. PROCESS generates distance/time coordinates for each aircraft for the specific altitude of level-off. The input data, as well as the generated distance/time coordinates, are output. If the lethal overpressure and/or thermal radius and the time of shock arrival are not present in the input, PROCESS computes these values for the yield of each type of missile. The nuclear effects information is summarized in the output from PROCESS.
DETAREA	Called by QUANTO. From given aircraft profiles and geometry of flyout and detonation, DETAREA computes the lethal area with respect to the moving aircraft, i.e., the circle/petal area describing the thermal/overpressure kill region.
ALOUT	Called by QUANTO. ALOUT produces a list of the allocation n_{ij} in two formats. First, by target: the missiles allocated to that target are listed in order by submarine number and salvo number within the submarine. Second, by submarine: the missiles are listed in order by salvo, together with the targets to which they are allocated.

Table III (cont'd)

Routine Name	Description
ALINT	Called by QUANTO.
	ALINT integerizes the allocation matrix n_{ij}. This process is not a simple rounding of the nonintegral n_{ij} values, but an assignment of integral values to the highest fractional parts so as to make $\sum_{i=j}^{M} n_{ij} = N_j$ for each j.
VINT	Called by QUANTO.
	VINT integerizes the beddown values V_i in a manner similar to ALINT.
TGTKIL	Called by QUANTO.
	TGTKIL computes the survivability products $\prod_{j=1}^{L} S_{ij}^{n_{ij}}$ (for each aircraft type), the λ_{ij} values, the number of aircraft killed at each base, and a rough idea (obtained by rounding n_{ij} values) of the number of weapons allocated to each base.
ADJLAM	Called by QUANTO.
	ADJLAM finds Δn by first finding $\lambda_{k\ell} = \min_i \{\lambda_{i\ell}$ such that $n_{i\ell} \geq 0.0001\}$ $i = 1, 2, \ldots, M$, and $\lambda_{m\ell} = \max_i \{\lambda_{i\ell}\}$ $i = 1, 2, \ldots, M$, such that $\lambda_{k\ell} < \lambda_{m\ell} - \varepsilon$ for some weapon group ℓ and some tolerance ε. Then ADJLAM either computes the proper allocation adjustment Δn to force the value of $\lambda_{k\ell}$ toward the value of $\lambda_{m\ell}$ (for the single type aircraft model) or calls the function XNEWT to compute Δn (for the mixed force model). Tallies of weapons allocated to each base are updated after the change of n_{ij} by the Δn adjustments.

Table III (cont'd)

Routine Name	Description
SUBADJ	Called by QUANTO. SUBADJ locates the least effective submarine and the submarine location with the most potential, as described in the test on submarine optimization. The submarine is relocated to t e better position and its missiles are allocated to bases having high λ_{ij} values, as described in the text.
XAREA	Called by QUANTO. XAREA computes the area of intersection of a circle and an annulus, under all conditions of annulus radii, circle radius, and offset of circle center.

SECTION V

ASSUMPTIONS

In this section, the principal assumptions in QUANTO are described and are briefly discussed.

Assumption: All input SLBMs are used against aircraft, i.e., the attacking force decides what portion of its SLBMs to use against the flushing aircraft force prior to running a problem and the SLBMs in QUANTO represent that portion. Of course, for each submarine, only a partial load of missiles need be input for a problem.

Assumption: The survival probabilities, S_{ij}, are independent. No collateral damage may affect an aircraft departing one base as a result of a detonation of an SLBM allocated to another base. Furthermore, the effectiveness of a weapon in group j on base i is measured by S_{ij}, a numerical quantity which is independent of the number of weapons which have previously arrived or will subsequently arrive. Stated differently, the area in which aircraft may be located at the time of a later weapon arrival is not considered to contain voids left by previously arriving weapons. Detailed Monte Carlo simulation of QUANTO-produced attacks shows that QUANTO's predicted kill (using such S_{ij} values) is close to the actual kill resulting from the simulation.

Assumption: Thermal and overpressure effects have lethality according to a cookie-cutter criterion. In other words, an aircraft with hardness of x psi and y cal/cm^2 is killed if it experiences either of these levels or higher, but is safe from $(x-\varepsilon)$ psi and $(y-\varepsilon)$ cal/cm^2 for any $\varepsilon > 0$, no matter how small.

Assumption: At all times, aircraft are uniformly distributed within a maximum circle, defined by the first aircraft's range, the area of which is continually increasing with time. Thus, the survivabilities S_{ij} are computed assuming the attacker will pattern his weapons for uniform coverage of the maximum circle of aircraft. In some cases (few aircraft at early weapon arrival times), this assumption is modified to allow computation of S_{ij} by assuming instead that the aircraft are uniformly distributed throughout an annulus. In this way, a lower

computed S_{ij} results and more realistic expected kills result. Of course, if the aircraft do not disperse in a circular pattern, and the attacker were to be granted advance knowledge of these flyout tactics, a greater expected kill would result since the weapons have a smaller area to attack.

Assumption: The aircraft radially emanate from a point called the centroid of the aircraft which, for a given flyout profile and turn geometry, may be determined. The distance of the centroid from brake release point is a parameter which may be input for each base, it is based on the distance the aircraft flies without turning while raising its gear, climbing to turn altitude, etc.

Assumption: In the computation of S_{ij}, the detonation point of the weapon is at one of several places, as described in another portion of this report. The assumption of detonation point is such as to be in agreement with the uniform-attack-of-the-aircraft-area assumption, with a modification of the location (1) when protrusion of the lethal area beyond the maximum circle occurs, (2) when an annular P_k computation yields a better estimate of S_{ij}, or (3) when a weapon on the centroid kills all aircraft of a single type. In this way, some pains are taken to compute S_{ij} based on a reasonable estimate of the weapon location, without knowledge of where other weapons are allocated.

Assumption: When multiple aircraft types are included in the model, all aircraft radially emanate from a single centroid and weapons are patterned to attack uniformly the area of all aircraft types. Point values of aircraft of different types may make some bases more attractive than others; but on those bases, the attack is assumed to be uniform.

SECTION VI

LAGRANGE MULTIPLIERS IN THE MIXED FORCE ALLOCATION PROBLEM

When more than one type of aricraft may be leaving each base, the problem of determining the optimal missile laydown is more complicated than the previously described model. For purposes of explanation, the following will assume two aircraft types, bombers and tankers, indicated by B and T subscripts, respectively. However, the procedures are general for any number of aircraft types. The objective to be maximized in the mixed force allocation problem is

$$f(n_{ij}) = \sum_{i=1}^{M} V_{iB}\left[1 - \prod_{j=1}^{L} S_{ijB}{}^{n_{ij}}\right] + \sum_{i=1}^{M} V_{iT}\left[1 - \prod_{j=1}^{L} S_{ijT}{}^{n_{ij}}\right] \tag{27}$$

with the same stockpile constraints

$$\sum_{j=1}^{M} n_{ij} = N_j,\ i = 1, 2, \ldots, L$$

The function $f(n_{ij})$ is now the expected kill value of both bombers and tankers with a sum over the target index i for each type of aircraft. The values of the bombers and tankers, respectively, leaving base i are V_{iB} and V_{iT}, where each type aircraft may be worth a different amount of value per aircraft. The survivabilities S_{ijB} and S_{ijT} of the bombers and tankers, respectively, of target i from a weapon in group j, must be computed slightly differently than the previous S_{ij}.

Since the types of aircraft may have different thermal and overpressure hardness levels, there is a circle/petal combination for each aircraft type at a single weapon detonation point. Using the same notations as before, with the additional B (bomber) and T (tankers) subscripts (following the slashes), the following formulas for $P_{k/B}$ and $P_{k/T}$ are used for the computation of bomber and tanker P_k's.

In the mixed force allocation problem, it will be assumed that the SLBMs are aimed uniformly at the entire area of all aircraft (of all types). When the geometry is such that the tankers are within the bombers, as shown in figure 13,

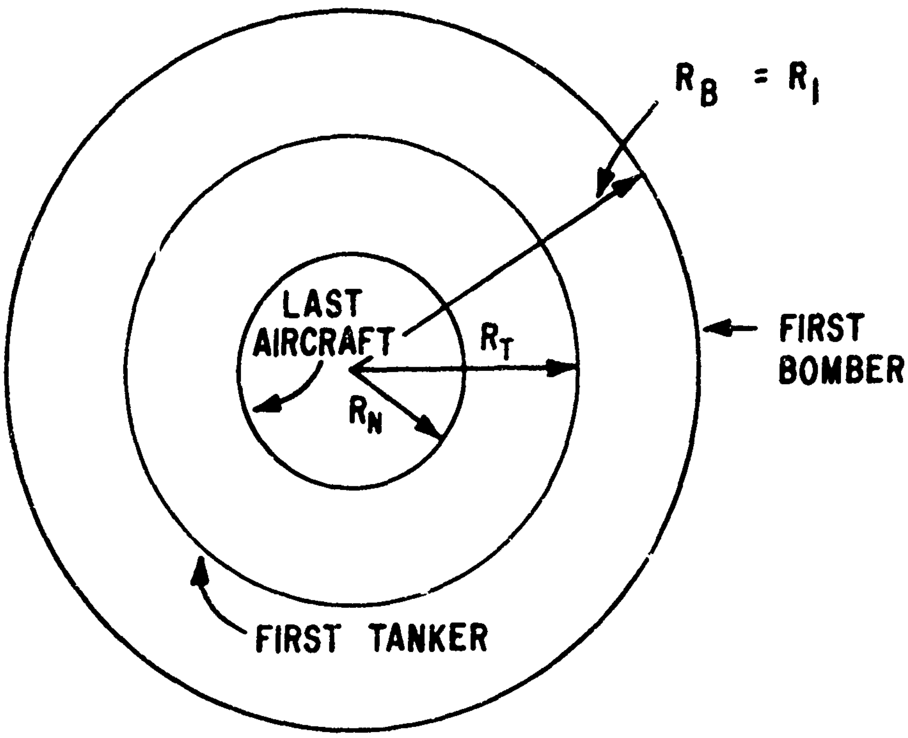

Figure 13. Bomber/Tanker Mix

the $P_{k/T}$ may be approximated by the product of (1) the probability that a random placement within the bomber circle lands within the tanker circle, and (2) the probability of kill of a tanker given that the weapon detonates within the tanker circle. If $A_{L/T}$ represents the lethal area of the SLBM against tankers, the product is

$$P_{k/T} = \frac{\pi R_T^2}{\pi R_B^2} * \frac{A_{L/T}}{\pi R_T^2} = \frac{A_{L/T}}{\pi R_B^2} \tag{28}$$

where overlap of the lethal area over the circle of radius R_T has been ignored. Thus, for these assumptions and approximations, $P_{k/T}$ is independent of R_T. Should the bombers be within the tankers,

$$P_{k/B} = \frac{A_{L/B}}{\pi R_T^2} \tag{29}$$

so the maximum radius always appears in the denominator.

For the mixed aircraft model, let R_1 and R_N represent the radial distances of the most and least distant aircraft, respectively, from the single centroid (the same point for all aircraft types) at the time of weapon arrival. The subscript MAX means the largest for all aircraft types, and other notations are analogous to those used in the discussion of the single type aircraft model.

A. If

$$R_1 \leq R_{LR/MAX} \text{ [for 0]} \tag{30}$$

then

$$P_{k/T} = \min \left\{ 1, \frac{\pi\left(R^2_{LR/T} - R_N^2\right)}{\pi\left(R_1^2 - R_N^2\right)} \right\} \tag{31}$$

and

$$P_{k/B} = \min \left\{ 1, \frac{\pi\left(R^2_{LR/B} - R_N^2\right)}{\pi\left(R_1^2 - R_N^2\right)} \right\} \tag{32}$$

assuming that the weapon detonates at the centroid.

B. If

$$R_1 > R_{LR/MAX} \text{ [for 0]}$$

and

$$R_1/\sqrt{2} + R_{LR/MAX} \left[\text{for } R_1/\sqrt{2}\right] \leq R_1 \tag{33}$$

so that neither circle/petal lethal area protrudes beyond R_1, then

$$\text{(1)} \quad P_{k/T} = \min \left\{ 1, \frac{A_{L/T} \left[\text{for } R_1/\sqrt{2}\right]}{\pi R_1^2} \right\} \tag{34}$$

and

$$P_{k/B} = \min \left\{ 1, \frac{A_{L/B} \left[\text{for } R_1/\sqrt{2}\right]}{\pi R_1^2} \right\} \tag{35}$$

if either

$$\frac{A_{LAN/T}\left[\text{for }\left(R_1 + R_N\right)/2\right]}{\pi\left(R_1^{\,2} - R_N^{\,2}\right)} > \frac{A_{L/T}\left[\text{for }R_1/\sqrt{2}\right]}{\pi R_1^{\,2}} \tag{36}$$

or

$$\frac{A_{LAN/B}\left[\text{for }\left(R_1 + R_N\right)/2\right]}{\pi\left(R_1^{\,2} - R_N^{\,2}\right)} > \frac{A_{L/B}\left[\text{for }R_1/\sqrt{2}\right]}{\pi R_1^{\,2}} \tag{37}$$

Otherwise,

$$(2)\quad P_{k/T} = \frac{A_{LAN/T}\left[\text{for }\left(R_1 + R_N\right)/2\right]}{\pi\left(R_1^{\,2} - R_N^{\,2}\right)} \tag{38}$$

and

$$P_{k/B} = \frac{A_{LAN/B}\left[\text{for }\left(R_1 + R_N\right)/2\right]}{\pi\left(R_1^{\,2} - R_N^{\,2}\right)} \tag{39}$$

In words, if the annulus P_k computations in equation (38) above are both smaller than their corresponding circular P_k computations in equation (34), then the annulus P_k formulas are used for all aircraft types. Otherwise, the formulas in equation (34) are used.

C. Finally, if either circle/petal protrudes when positioned at $R_1/\sqrt{2}$, the weapon is moved toward the centroid as before and

$$(1)\quad P_{k/T} = \frac{A_{L/T}\left[\text{for }R_1 - R_{LR/MAX}\left[\text{for }R_1/\sqrt{2}\right]\right]}{\pi R_1^{\,2}} \tag{40}$$

and

$$P_{k/B} = \frac{A_{L/B}\left[\text{for }R_1 - R_{LR/MAX}\left[\text{for }R_1/\sqrt{2}\right]\right]}{\pi R_1^{\,2}} \tag{41}$$

if either

$$\frac{A_{LAN/T}\left[\text{for }\left(R_1 + R_N\right)/2\right]}{\pi\left(R_1^{\,2} - R_N^{\,2}\right)} > \frac{A_{L/T}\left[\text{for } R_1 - R_{LR/MAX}\left[\text{for } R_1/\sqrt{2}\right]\right]}{\pi R_1^{\,2}} \tag{42}$$

or

$$\frac{A_{LAN/B}\left[\text{for }\left(R_1 + R_N\right)/2\right]}{\pi\left(R_1^{\,2} - R_N^{\,2}\right)} > \frac{A_{L/T}\left[\text{for } R_1 - R_{LR/MAX}\left[\text{for } R_1/\sqrt{2}\right]\right]}{\pi R_1^{\,2}} \tag{43}$$

Otherwise,

$$(2)\quad P_{k/T} = \frac{A_{LAN/T}\left[\text{for }\left(R_1 + R_N\right)/2\right]}{\pi\left(R_1^{\,2} - R_N^{\,2}\right)} \tag{44}$$

and

$$P_{k/B} = \frac{A_{LAN/B}\left[\text{for }\left(R_1 + R_N\right)/2\right]}{\pi\left(R_1^{\,2} - R_N^{\,2}\right)} \tag{45}$$

It should be noted that these formulas have been written in a slightly different form to ensure that the P_k computations for different aircraft types are all based on the same placement of the weapon.

The survivabilities are simply

$$S_{ijB} = 1 - P_{k/B} * (\text{reliability of weapon in group } j)$$

$$S_{ijT} = 1 - P_{k/T} * (\text{reliability of weapon in group } j)$$

The technique for solution of the constrained maximization problem for the mixed aircraft force is very similar to the techniques for the previous model. The Lagrangian function for the new objective function $f(n_{ij})$ is

$$h\left(n_{ij}, \lambda_j\right) = f\left(n_{ij}\right) + \sum_{j=1}^{L} \lambda_j \left[\sum_{i=1}^{M} n_{ij} - N_j\right] \tag{46}$$

Setting the partial derivatives equal to zero, as before, yields

$$\frac{\partial h}{\partial n_{k\ell}} = \lambda_{k\ell B} + \lambda_{k\ell T} + \lambda_\ell = 0$$

$$k = 1, 2, \ldots, M;\ \ell = 1, 2, \ldots, L \tag{47}$$

where

$$\lambda_{k\ell B} \equiv -V_{kB} \left(\ell n\ S_{k\ell B}\right) \prod_{j=1}^{L} S_{kjB}^{\ n_{kj}} \tag{48}$$

and

$$\lambda_{k\ell T} \equiv -V_{kT} \left(\ell n\ S_{k\ell T}\right) \prod_{j=1}^{L} S_{kjT}^{\ n_{kj}} \tag{49}$$

For the mixed aircraft model, the new definition of $\lambda_{k\ell}$ is

$$\lambda_{k\ell} \equiv \lambda_{k\ell B} + \lambda_{k\ell T} \tag{50}$$

or in the case of more than two aircraft types, $\lambda_{k\ell}$ would be the sum of the lambdas corresponding to each aircraft type. Fixing ℓ and letting k vary results in the system of equations

$$\lambda_{k\ell} = -\lambda_{\ell},\ k = 1,\ 2,\ \ldots,\ M$$

which has the same appearance as in the single aircraft model, although $\lambda_{k\ell}$ is differently defined.

The iterative procedure is again based on finding $\lambda_{k\ell} < \lambda_{m\ell}$ with $n_{k\ell} \geq 0.0001$, and choosing Δn so that the new values of $\lambda_{k\ell}$ and $\lambda_{m\ell}$, say, $\hat{\lambda}_{k\ell}$ and $\hat{\lambda}_{m\ell}$, become equal. This value of Δn is the root of

$$\begin{aligned} g(\Delta n) &\equiv \hat{\lambda}_{k\ell} - \hat{\lambda}_{m\ell} \\ &= \hat{\lambda}_{k\ell B} + \hat{\lambda}_{k\ell T} - \hat{\lambda}_{m\ell B} - \hat{\lambda}_{m\ell T} \\ &= S_{k\ell B}^{\ -\Delta n}\ \lambda_{k\ell B} + S_{k\ell T}^{\ -\Delta n}\ \lambda_{k\ell T} - S_{m\ell B}^{\ +\Delta n}\ \lambda_{m\ell B} - S_{m\ell T}^{\ +\Delta n}\ \lambda_{m\ell T} \end{aligned} \tag{51}$$

The root of Δn^* of $g(\Delta n)$ is found by Newton's successive approximation method, where

$$\Delta n_{i+1} = \Delta n_i - \frac{g(\Delta n_i)}{g'(\Delta n_i)},\ i = 1,\ 2,\ \ldots. \tag{52}$$

In QUANTO, when $|\Delta n_{i+1} - \Delta n_i| < \varepsilon$ (the same tolerance used in the test for convergence of $\lambda_{k\ell}$ and $\lambda_{m\ell}$), Δn^* is set equal to Δn_{i+1}. This iterative formula may diverge (as tested by $|\Delta n_{i+1}| \geq 200$ in QUANTO) for a given selection of Δn_o. Convergence is attempted for Δn_o selections of $n_{k\ell}$, 0, and $n_{k\ell}/2$, successively, until the iteration successively converges. If the root $\Delta n^* > n_{k\ell}$, Δn is chosen as $n_{k\ell}$; i.e.,

$$\Delta n = \min \left\{\Delta n^*, n_{k\ell}\right\} \tag{53}$$

so as to keep $(n_{k\ell} - \Delta n)$ nonnegative.

This choice of Δn results in the maximum increase in the expected kill. The proof of this statement is quite similar to the analogous proof in the single aircraft case. If the kill contribution to the objective function $f(n_{ij})$ from targets k and m after Δn weapons are moved from target k to target m is indicated as $r(\Delta n)$, then the unconstrained maximum of $r(\Delta n)$ occurs where $r'(\Delta n) = 0$. But this is at Δn^*, the root of $g(\Delta n)$, since

$$r'(\Delta n) = -g(\Delta n) \tag{54}$$

The maximum of $r(\Delta n)$, constrained by $0 \leq \Delta n < n_{k\ell}$, occurs at $\Delta n = n_{k\ell}$ if $\Delta n^* > n_{k\ell}$, since

$$r'(0) = \hat{\lambda}_{m\ell} - \hat{\lambda}_{k\ell} > 0$$

and

$$r'' = -g'(\Delta n) \leq 0$$

The iterative procedure for the mixed aircraft model is exactly the same as that for the single aircraft model, with the exception of the new definition of $\lambda_{k\ell}$ and the Newton procedure for finding Δn^*. The submarine relocation procedure is also based on the new $\lambda_{k\ell} \equiv \lambda_{k\ell B} + \lambda_{k\ell T}$.

The beddown optimization procedure is slightly changed in that each beddown change simultaneously moves some of each type of aircraft. Thus, for bombers, the value

$$\Delta V_B = V_{kB}^* \left(\prod_{j=1}^{L} S_{mjB}^{\ n_{mj}} - \prod_{j=1}^{L} S_{kjB}^{\ n_{kj}} \right) \tag{55}$$

is shifted from base k to base m where these bases have, respectively, the lowest and highest survivability products

$$\prod_{j=1}^{L} S_{ijB}^{\;n_{ij}}$$

but only those bases k for which $V_{kB} > 0.0001$ compete for the lowest product. The value of tankers shifted, ΔV_T, (computed like ΔV_B, replacing "B" subscripts with "T" subscripts) depends on the tanker survivabilities S_{ijT} and values V_{iT}, and thus, bomber and tanker relocations may involve different pairs of bases. The beddown optimization stops when the total number of aircraft (of all types) to be moved in a single beddown change is less than 0.05.

SECTION VII

BENEFITS AND FUTURE USES OF QUANTO

QUANTO was developed to investigate the sensitivity of total bomber force survivability to variations in thermal and overpressure hardness. In the process of this development, numerous other parameters have been included as variables in the model. Consequently, the model is useful for evaluating the sensitivity of surviving aircraft to changes in aircraft beddown and flight profiles, numbers and types of submarines or missiles, SLBM performance characteristics, and reaction times, as well as aircraft hardness. Alternative missile laydowns, submarine locations, and aircraft beddowns may be compared and evaluated using QUANTO to compute expected kills. Contractual studies may be evaluated for validity and contrasted with QUANTO to aid in understanding their results. In-house and intra-AF investigations are facilitated by the availability of QUANTO.

QUANTO has a great deal of flexibility. It is relatively fast and easy to use compared to other flush models. An optimal laydown may be computed in 1 to 3 minutes of computer time, submarines may be optimized in about 5 minutes, and optimal beddowns require up to 10 minutes, where these times are largely dependent on the quality of the selection of initial laydowns, submarine positions, and beddowns. Each weapon is considered a separate entity, not as a member of one of a fixed set of predefined patterns. In addition, QUANTO permits multiple types of aircraft and SLBMs, each with its own performance characteristics. The modular construction of QUANTO permits investigation of selective changes in the assumptions upon which the model is based, with selective program changes.

The projected future uses of QUANTO include the evaluation of other models and results of flush studies, studies of the effects of parametric variations on the survivability of a mixed force, and in-house experimentation and sensitivity analysis.

APPENDIX I

APPLICATIONS OF LAGRANGE MULTIPLIERS

The Basic Problem

The computer program called QUANTO uses the Lagrange multiplier method to optimize the allocation of weapons. This appendix is provided to introduce the reader to the basic fundamentals of the technique.

The problem:

Maximize $f\left(x_1, x_2, \ldots, x_n\right)$

Subject to $g_j\left(x_1, x_2, \ldots, x_n\right) = b_j,\ j = 1, 2, \ldots, m;\ m < n$

Lagrange method:

Form the function:

$$h\left(x_1, x_2, \ldots, x_n, \lambda_1, \lambda_2, \ldots, \lambda_m\right) \equiv f\left(x_1, x_2, \ldots, x_n\right) + \sum_{j=1}^{m} \lambda_j \left\{g_j\left(x_1, x_2, \ldots, x_n\right) - b_j\right\}$$

where the λ_j are constants (known as Lagrange multipliers) as yet to be determined in value. Note that when the constraints are satisfied, h is formed merely by adding multipliers of zeros to f. Now treat x_i, $i = 1, 2, \ldots, n$, as independent variables, and write down the conditions

$$\frac{\partial h}{\partial x_1} = 0$$

$$\frac{\partial h}{\partial x_2} = 0$$

$$\vdots$$

$$\frac{\partial h}{\partial x_n} = 0$$

$$\left.\begin{array}{l} \dfrac{\partial h}{\partial \lambda_1} = 0 \\[2ex] \dfrac{\partial h}{\partial \lambda_2} = 0 \\ \quad\cdot \\ \quad\cdot \\ \quad\cdot \\ \dfrac{\partial h}{\partial \lambda_m} = 0 \end{array}\right\} \begin{array}{l}\text{Constraint} \\ \text{Equations}\end{array}$$

Solving the (n + m) equations for the x_i and λ_j will yield the critical points of f.

EXAMPLE 1:

Minimize $f(x,y,z) = x^2 + y^2 + z^2$

subject to the conaition that (x,y,z) is on the plane

$$S = \{(x,y,z): \ 2x + 3y - z - 1 = 0\}$$

First introduce the new variable λ to form

$$F(x,y,z,\lambda) = (x^2 + y^2 + z^2) + \lambda(2x + 3y - z - 1)$$

Now compute F_x, F_y, F_z, and F_λ.

$$F_x = 2x + 2\lambda = 0$$

$$F_y = 2y + 3\lambda = 0$$

$$F_z = 2z - \lambda = 0$$

$$F_\lambda = 2x + 3y - z - 1 = 0$$

These equations yield

$$x = \frac{1}{7}, \ y = \frac{3}{14}, \ z = -\frac{1}{14}, \ \lambda = -\frac{1}{7}$$

The solution satisfies $F_\lambda = 0$ and is, therefore, on the plane

$$2x + 3y - z - 1 + 0$$

EXAMPLE 2:

Maximize $f(A,B) = 6A + 2B + AB - A^2 - 2B^2 + 5$

subject to $p(A,B) = 2A - B = 8$

Solve by finding the (local) optimum of

$$h(A,B,\lambda) \equiv f(A,B) + \lambda[p(A,B) - 8]$$

$$= 6A + 2B + AB - A^2 - 2B^2 + 5 + \lambda\ (2A - B - 8)$$

Set partial derivatives to zero.

$$\frac{\partial h}{\partial A} = 6 + B - 2A + 2\lambda = 0$$

$$\frac{\partial h}{\partial B} = 2 + A - 4B - \lambda = 0$$

$$\frac{\partial h}{\partial \lambda} = 2A - B - 8 = 0$$

Solving yields

$$A = \frac{33}{7}$$

$$B = \frac{10}{7}$$

$$\lambda = 1$$

Thus

$$f\left(\frac{33}{7}, \frac{10}{7}\right) = 16.5714$$

Writing the constraint equation as $2A - B = \delta$ gives

$$\frac{\partial h}{\partial \delta} = -\lambda = -1$$

One can then see that increasing δ has the effect of decreasing h (and, therefore, f) at the rate of -1 unit of f per unit of δ. Indeed, regardless of what h looks like, if the constraint is written as $p = \delta$, then $\frac{\partial h}{\partial \delta}$ will always equal $-\lambda$.

Examples one and two are both performed in the same manner even though one is a maximum and the other is a minimization problem. The manner in which one would differentiate between which has occurred is by calculating the Hessian matrix.

APPENDIX II

QUANTO'S ITERATIVE PROCEDURE
Three Examples

This appendix is provided for the reader to become acquainted with the types of problems solved by QUANTO. The three examples serve to illustrate the three basic options available to the using organization. The type problems addressed are

1. Optimize n_{ij}

   Given: three targets
   three weapon groups

2. Optimize aircraft beddown

   Given: bomber/tanker mix
   ten tankers
   seven bombers

3. Optimize submarine locations

In cases 1 and 3 the optimal n_{ij} is found prior to the optimization of the aircraft beddown or submarine locations.

EXAMPLE 1

Suppose L = 3, M = 3, i.e., there are three targets and three weapon groups. Let the number of weapons in each group be $N_1 = 4$, $N_2 = 3$, $N_3 = 7$ and suppose

$$S_{ij} = \begin{bmatrix} 0.8 & 0.7 & 0.9 \\ 0.6 & 0.5 & 0.7 \\ 0.2 & 0.1 & 0.3 \end{bmatrix}$$

This matrix represents the survival probabilities of target i from one weapon in group j, e.g., the survival probability of target 2 from a single weapon in group 2 is 0.5. Let V_1, V_2, V_3 = 10, 5, 2, respectively. As the first step, an arbitrary allocation is formed, and suppose we choose

$$\left(n_{ij}\right)_1 = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}$$

where, for example, it was decided to send three weapons from the 7 in group 3 to target 1. The λ matrix for this (n_{ij}) is

$$\begin{bmatrix} -V_1 \ln S_{11} \prod_{j=1}^{3} S_{1j}^{\,n_{1j}} & -V_1 \ln S_{12} \prod_{j=1}^{3} S_{1j}^{\,n_{1j}} & -V_1 \ln S_{13} \prod_{j=1}^{3} S_{1j}^{\,n_{1j}} \\ -V_2 \ln S_{21} \prod_{j=1}^{3} S_{2j}^{\,n_{2j}} & -V_2 \ln S_{22} \prod_{j=1}^{3} S_{2j}^{\,n_{2j}} & -V_2 \ln S_{23} \prod_{j=1}^{3} S_{2j}^{\,n_{2j}} \\ -V_3 \ln S_{31} \prod_{j=1}^{3} S_{3j}^{\,n_{3j}} & -V_3 \ln S_{32} \prod_{j=1}^{3} S_{3j}^{\,n_{3j}} & -V_3 \ln S_{33} \prod_{j=1}^{3} S_{3j}^{\,n_{3j}} \end{bmatrix}$$

and the numbers compute to be

$$\left(\lambda_{ij}\right)_1 = \begin{bmatrix} 0.51 & 0.82 & 0.24 \\ 0.38 & 0.51 & 0.2 \\ 0.06 & 0.08 & 0.04 \end{bmatrix}$$

The smaller λ's associated with target 3 indicate that weapons have been over allocated there. Starting with column 1, i.e., the weapons in group one,

$$\lambda_{31} < \lambda_{21}$$

so that

$$\Delta n = \frac{\ln\left(\frac{0.06}{0.38}\right)}{\ln\,(0.6)(0.2)} = 0.88$$

This adjustment of weapons from target 3 to target 2 will increase the objective function, and will equate λ_{31} and λ_{21}. The new (n_{ij}) is

$$\left(n_{ij}\right)_2 = \begin{bmatrix} 2.00 & 2 & 3 \\ 1.88 & 1 & 2 \\ 0.12 & 0 & 2 \end{bmatrix}$$

and

$$\left(\lambda_{ij}\right)_2 = \begin{bmatrix} 0.51 & 0.82 & 0.24 \\ 0.24 & 0.32 & 0.17 \\ 0.24 & 0.34 & 0.18 \end{bmatrix}$$

Since

$$\lambda_{22} < \lambda_{12}$$

$$\Delta n = \frac{\ell n \frac{0.32}{0.82}}{\ell n\left[(0.7)(0.5)\right]} = 0.88$$

and

$$\left(n_{ij}\right)_3 = \begin{bmatrix} 2.00 & 2.88 & 3 \\ 1.88 & 0.12 & 2 \\ 0.12 & 0.00 & 2 \end{bmatrix}$$

and

$$\left(\lambda_{ij}\right)_3 = \begin{bmatrix} 0.37 & 0.60 & 0.17 \\ 0.44 & 0.60 & 0.31 \\ 0.24 & 0.34 & 0.18 \end{bmatrix}$$

Continuing in this manner, the Lagrange multipliers $\lambda_{i\ell}$ corresponding to positive $n_{i\ell}$ will converge to the unique λ_ℓ for each weapon group ℓ. The final Lagrange multiplier matrix is

$$\left(\lambda_{ij}\right) = \begin{bmatrix} 0.34 & 0.55 & 0.16 \\ 0.34 & 0.46 & 0.24 \\ 0.32 & 0.45 & 0.24 \end{bmatrix}$$

which arises from the optimal allocation

$$\left(n_{ij}\right) = \begin{bmatrix} 4 & 3 & 0 \\ 0 & 0 & 5 \\ 0 & 0 & 2 \end{bmatrix}$$

which will give an expected target value return of 14.6 out of 17, the largest possible. This procedure is easily programmable. For very large matrixes (on the order of several thousand targets) there are more efficient procedures to adjust the multipliers by examing the convergence rates. For smaller cases on the order of a few hundred targets and weapons, the method above should not involve excessive computer time.

EXAMPLE 2

Suppose there are ten tankers and seven bombers, with a bomber twice the value of a tanker, bedded down as follows:

$$V_{iB} = \begin{bmatrix} 8 \\ 4 \\ 2 \end{bmatrix} \qquad V_{iT} = \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}$$

and suppose further that $N_1 = 4$, $N_2 = 3$, $N_3 = 7$ with

$$S_{ijB} = \begin{bmatrix} 0.8 & 0.7 & 0.9 \\ 0.6 & 0.5 & 0.7 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}, \quad S_{ijT} = \begin{bmatrix} 0.7 & 0.6 & 0.8 \\ 0.5 & 0.4 & 0.6 \\ 0.2 & 0.1 & 0.3 \end{bmatrix}$$

where the S_{ij}'s are determined by SLBM yield, reliability, trajectory, and aircraft vulnerability, takeoff profile and sequence. We start by making an initial guess at the allocation of SLBMs to bases as follows:

$$n_{ij} = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}$$

Recalling that

$$\lambda_{ijB} = -V_{iB} \left(\ell n\ S_{ijB}\right) \prod_j S_{ijB}^{\ n_{ij}}$$

and

$$\lambda_{ijT} = -V_{iT} \left(\ell n\ S_{ijT}\right) \prod_j S_{ijT}^{\ n_{ij}}$$

$$\lambda_{ij} = \lambda_{ijB} + \lambda_{ijT}$$

$$\lambda_{ijT} = \begin{bmatrix} 0.408 & 0.652 & 0.193 \\ 0.300 & 0.407 & 0.210 \\ 0.116 & 0.155 & 0.088 \end{bmatrix}$$

$$\lambda_{ijT} = \begin{bmatrix} 0.161 & 0.231 & 0.101 \\ 0.200 & 0.264 & 0.147 \\ 0.029 & 0.041 & 0.022 \end{bmatrix}$$

$$\lambda_{ij} = \begin{bmatrix} 0.569 & 0.883 & 0.294 \\ 0.500 & 0.671 & 0.357 \\ 0.145 & 0.196 & 0.110 \end{bmatrix}$$

We operate on this matrix column by column, choosing first the column having the largest difference in λ's. In column 2, $\lambda_{12}-\lambda_{32}$ represents this largest difference. The procedure requires a certain part of the weapons in group 2 to be moved from target 3 to target 1, since $\lambda_{32} < \lambda_{12}$. Since, however, there are no weapons in group 2 allocated to target 3 by our first (n_{ij}) guess, we need to look further. The next largest difference is $\lambda_{11}-\lambda_{31}$, and we move Δn weapons in group 1 from target 3 to target 1, where Δn is a root of

$$g(\Delta n) = S_{11B}^{-\Delta n}\ \lambda_{11B} + S_{11B}^{-\Delta n}\ \lambda_{11T} - S_{31B}^{+\Delta n}\ \lambda_{31B} - S_{31T}^{+\Delta n}\ \lambda_{31T}$$

This equation is solved by use of the Newton successive approximation method of root finding, where

$$\Delta n_{k+1} = \Delta n_k - \frac{g\left(\Delta n_k\right)}{g'\left(\Delta n_k\right)}$$

and in

$$(0.8)^{-\Delta n}(0.408)+(0.7)^{-\Delta n}(0.161)-(0.3)^{\Delta n}(0.116)-(0.2)^{\Delta n}(0.029) = 0$$

$$\Delta n = 0.88$$

Since $g'(\Delta n) > 0$ for $\Delta n \geq 0$, $g(\Delta n)$ has a unique solution. Therefore, our new allocation is

$$\left(n_{ij}\right) = \begin{bmatrix} 2.88 & 2 & 3 \\ 1.00 & 1 & 2 \\ 0.12 & 0 & 2 \end{bmatrix}$$

and the new λ_{ij} matrix is

$$\lambda_{ij} = \begin{bmatrix} 0.453 & 0.705 & 0.232 \\ 0.500 & 0.671 & 0.357 \\ 0.453 & 0.617 & 0.343 \end{bmatrix}$$

Notice that our choice of Δn forces $\lambda_{11} = \lambda_{31}$, which in turn increases the value killed by the SLBM attack. This procedure is repeated until the differences in the λ's become very small, or it becomes impossible to increase by shifting weapons. After six iterations, the final allocation is

$$\left(n_{ij}\right) = \begin{bmatrix} 3 & 3 & 1 \\ 1 & 0 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

and the value destroyed is about 21.3 out of the total 24. The bombers killed turn out to be 5.9 out of 7 and tankers 9.4 out of 10. An important question is: can the bombers and tankers be bedded down so as to decrease the number of kills to a minimum? As shown before, the minimum damage that can be inflicted by the SIBM attack occurs when the

$$\prod_{j=1}^{L} S_{ijB}^{\;n_{ij}}, \; i = 1, \ldots, N$$

are equal and when

$$\prod_{j=1}^{L} S_{ijT}^{\;n_{ij}}, \; i = 1, \ldots, N$$

are equal. In the above example,

$$\prod_{j} S_{ijB}^{\;n_{ij}}$$

are 0.17, 0.13, 0.12 and

$$\prod_{j} S_{ijT}^{\;n_{ij}}$$

are 0.07, 0.05, 0.06. These numbers indicate that the beddown is already a good one, and could only be slightly improved. Once the given n_{ij} matrix is in its final form, in this case with the λ_{ij} matrix converged to a tolerance of 0.001, the bombers and tankers can be shifted according to the formula

$$\Delta V = \left(\prod_{j=1}^{L} S_{mj}^{\;n_{mj}} - \prod_{j=1}^{L} S_{kj}^{\;n_{kj}} \right) V_k$$

applied to bomber and tanker values independently. With the given initial bomber and tanker values and a converged n_{ij} matrix (after 17 iterations)

$$\begin{bmatrix} 3.2649 & 3.0 & 0.0 \\ 0.7351 & 0.0 & 4.6912 \\ 0.0 & 0.0 & 2.3088 \end{bmatrix}$$

the products

$$\prod_{j=1}^{L} S_{ij}^{n_{ij}}$$

can be formed for each aircraft. Since there are three targets in this example, there are three resultant products. These are

$$\begin{array}{c} (1) \\ (2) \\ (3) \end{array} \begin{bmatrix} 0.1655 \\ 0.1289 \\ 0.1206 \end{bmatrix} \qquad \begin{bmatrix} 0.0674 \\ 0.0547 \\ 0.0621 \end{bmatrix}$$

Bombers Tankers

In computing the values to shift, ΔV, for each aircraft, the smallest product (where a value is present) is subtracted from the largest one and the difference is multiplied by the value on the target corresponding to the smallest product. This is the portion of value to be subtracted from the total value corresponding to the largest product. For example, for bombers, the difference in maximum and minimum product values is

$$0.1655 - 0.1206 = 0.0449$$

which when multiplied by the value corresponding to the lower product gives

$$\Delta V = 0.0449 \times 2 = 0.0898$$

Thus, the new bomber value matrix becomes

$$V_{iB} = \begin{bmatrix} 8.0898 \\ 4.0000 \\ 1.9102 \end{bmatrix}$$

By a similar process, the new tanker matrix becomes

$$V_{iT} = \begin{bmatrix} 5.0508 \\ 3.9492 \\ 1.0000 \end{bmatrix}$$

With these values, the laydown is again optimized. By moving the appropriate ΔV values iteratively from lower to higher $\Pi\, S_{ij}^{\;n_{ij}}$, we obtain the following beddown

$$V_{iB} = \begin{bmatrix} 12 \\ 2 \\ 0 \end{bmatrix} \qquad V_{iT} = \begin{bmatrix} 0 \\ 6 \\ 4 \end{bmatrix}$$

The new allocation of SLBMs changes only slightly as follows

$$n_{ij} = \begin{bmatrix} 4 & 3 & 0 \\ 0 & 0 & 5 \\ 0 & 0 & 2 \end{bmatrix}$$

and similarly the number of kills decreases only slightly to 5.9 out of 7 bombers and 9.3 out of 10 tankers, for a total value destroyed of 21.2. Note that although the SLBM allocation and total SLBM strike effectiveness changed very little, the beddown, in comparison, changed considerably. Hence, within the context of random events (i.e., SLBM reliabilities, CEPs, etc.) precipitating uncertainties as to exact numbers of bombers killed, the beddown problem does not appear to lend itself to a unique solution. Also, note that the bomber to tanker value ratio of 2:1 was taken as fixed for all bases; however, this value ratio can be varied from base to base. Also, note that by inserting zeros in the appropriate locations of the S_{ij} matrixes, we can effectively enforce the constraints of not permitting tankers or bombers to be based at particular locations.

EXAMPLE 3

This example demonstrates the capabilities of QUANTO on a small hypothetical problem which resembles those actually run with QUANTO. The input deck for this problem is listed in figure 14, in the format described in the user documentation of QUANTO. The discussion presented here will be in the form of a guide to reading the output, most of which is shown in appendix III.

The first two lines of output serve to uniquely identify the run and give basic problem data. Both the beddown and the submarine locations appear on the first page of output. Targets 1 to 4 are located at points in Colorado, North

```
     4    3     1     2     1     2     2     2     1     3
    38.25     103.25      0.0         2           15.0
1111111111111111
    48.25      97.50      5.5         1           15.0
1111111111111111
    42.85      91.40      0.0         2           15.0
1111111111111111
    35.7       85.90      5.5         1            5.0
11111
     1.0        4.5       1.5        60
      .1667      .08333
    36.75      74.0       1     2     1
    28.05      93.45      2     2     1
    45.65     126.05      2     2     1
      .25        .9        .95         .95      300.0     2000.0    1500.0
    14          4.3750  310.0        5.3190   520.0         6.430   810.0
     7.5417 1130.0        8.6528 1475.0         9.7639 1765.0
    10.8750 2050.0       11.9861 2310.0        13.0972 2530.0
    14.2083 2740.0       15.3195 2930.0        16.4305 3110.0
    17.5417 3260.0       17.9861 3290.0
 5000.0     2500.0
    20        PHANTOM PROBLEM PROFILE
      0.0                   0.0           0.0
   2000.0                  20.0           9.0          .1406
   3900.0                  30.0          30.0          .1406
   6200.0                  40.0          92.0          .1946
   9500.0                  50.0         360.0          .2538
  13400.0                  60.0         500.0          .3154
  15600.0                  66.0         500.01         .3549
  17300.0                  68.0         500.02         .3672
  17900.0                  69.0         500.03         .3734
  18200.0                  71.0         500.04         .3863
  22750.0                  78.0         500.05         .4355
  26800.0                  86.0         500.06         .4620
  33000.0                  94.0         500.07         .5636
  37000.0              101.0            500.08         .5955
  44000.0              109.0            500.09         .6379
  49750.0              117.0            500.10         .6617
  55900.0              125.0            500.11         .6865
  61000.0              132.0           1425.0          .7037
  68000.0              140.0           3950.0          .7275
  75500.0              148.0           5000.0          .7312     6.2
  7600.0                  .849           60.0          10.0
     0.0        1.0        0.3       10.            5.0    10000.0


     .01      20  100      .0001
    2     2
    3     2
    1     4
    2     2
    3     3
000000000000000000000
```

Figure 14. Sample Input Deck

Dakota, Iowa, and Tennessee and the submarine locations are about 100 nm off the coasts of Virginia, Louisiana, and Oregon. The targets are distinguished not only by their distances from the submarines but by the number of aircraft and the distances (in nm) to the centroid of the aircraft flyout pattern (a function of the numbers of runways). Note that aircraft can take off with small intervals from bases with dual runways. The missile parameters and trajectory data are given on pages 71 and 72 of appendix III. Page 73 lists the input aircraft flyout profile data and pages 74, 75, and 76 show the profile generated by QUANTO with the aircraft leveled-off at 5000 feet. The bottoms of pages 77 and 78 show the lethal radii and time of (overpressure) shock arrival for the aircraft of hardness indicated on page 71, along with other standard output from the nuclear effects routines.

A table of lethal areas (as a function of the distance Q of the detonation from the centroid) is then built as each change of distance to centroid (DSPT) is encountered in the target list. When the aircraft has not reached its terminal altitude (at distance DISMIN), new lethal radii are obtained based on the actual aircraft altitude after it has traveled a distance (Q + DSPT) from the centroid, as shown for two such values of Q on pages 79 to 82 and 85 to 88. Occasionally, a value of Q results in a geometry of lethal area in which the boundaries of the overpressure and thermal kill regions have multiple intersections. When this occurs, approximately two pages of indicators are output to enable a detailed study of this geometry. This output may be ignored on production runs. Lethal area tables appear on pages 83 and 89 of appendix III.

The lethal areas of SLBMs are based on assumed detonation points on each target, dependent on where the aircraft are at time of weapon arrival. Distances and missile flight times from submarines to targets and samples of the computed aircraft locations appear on pages 84 and 90 to 92. Note that weapons are numbered sequentially through the salvos of each submarine location, but only the first SLBM of each submarine's two salvos appears in this output in order to reduce the quantity of printout while providing enough information to indicate when each SLBM arrives on each target. Note also that the annulus P_k line is only printed occasionally; this is because it is not computed in instances in which the program knows beforehand that the circular P_k will be the smaller of the two P_ks. The computed S_{ij} values are listed on page 93 where i is the target number and j is the weapon number, but one row of output contains only the S_{ij} corresponding to SLBMs from a single submarine location.

The chart at the top of page 94 lists the base-by-base kill resulting from the input laydown (prior to optimization). Since each aircraft is worth one point of value, the number of aircraft and the value are equal. The convergence to the optimal laydown (n_{ij} values) produces the output on pages 94 to 109, where the long form of the output has been requested to show each Δn value, the expected kill after each shift of Δn, and the allocation and multipliers λ_{ij} after each convergence to the tolerances ε = 0.1, 0.01, 0.001, and 0.0001. The best nonintegral laydown appears on page 107, listed by target and then by submarine and salvo. The expected kill increased from 14.9765 to 19.7147 (out of a total of 50) aircraft during the convergence. Target-by-target kills and weapons allocated (if n_{ij} are rounded off) appear on page 108.

Optimization of submarine positioning has been requested in this example, and occurs prior to the requested beddown optimization. The first submarine move is indicated on page 110 and the resultant positions of all submarines appear on page 111 following the initial allocation (prior to convergence again). The convergence to the best laydown with the submarines in their new positions follows with the resultant kill shown on page 134. The first submarine move improved the kill (after convergence) from 19.7147 to 20.8742. The second submarine move is shown on pages 136 and 137. A summary of the submarine optimization appears below.

After Submarine Move No.	Converged Expected Kill	Page
0	19.7147	108
1	20.8742	134
2	21.2415	139
3	20.8742	144

After the third submarine move, the expected kill decreased slightly so the submarines are fixed (2 at point 1, 2 at point 2, and 1 at point 3) and beddown optimization begins with the first shift of aircraft on pages 145 and 146. Convergence to the optimal laydown follows each beddown change and is summarized on the following page.

After Value Shift No.	Converged Expected Kill	Page
0	20.8742	144
1	18.7694	148
2	16.8932	152
3	16.8171	156
4	16.5884	160
5	16.5863	164

Since the sixth value shift was to be a shift of less than 0.05 aircraft (see page 165 of appendix III), the beddown optimization was terminated. The problem is terminated by integerizing both the laydown and the beddown. The expected kill tends to decrease due to the laydown integerization and increase due to the beddown integerization. The resultant expected kill following both integerizations is 16.4428. The integral laydown appears on page 166 and the base-by-base kills and integral beddown appear on page 167.

In analyzing the output, a table of distances from submarine locations to targets is useful. These distances, to the nearest nautical mile, appear below and on pages 84 and 90 to 92.

Submarine Location		1	2	3
Target	1	1391	785	1107
	2	1241	1228	1174
	3	881	895	1489
	4	580	589	1902

The optimal laydown from the initial positioning of submarines appears on pages 107 and 108. It is interesting to note that the submarine at location 1 allocated its missiles to target 3, leaving target 4 to the submarines at location 2. Target 2 drew the most weapons even though it could not be hit as soon as the other targets. This was probably because target two's 15 aircraft departed from a single runway, and, therefore, were dispersing from a point 5.5 nm, in this case, from the brake release point. Dual runways permit more immediate dispersal since the aircraft can take off in opposite directions.

The optimization of the submarine positions resulted in only one submarine at location 3, even though this location had the best shot at target 2.

During the beddown optimization, the proximity of target 4 to the submarines at locations 1 and 2 made target 4 unattractive for bedding down aircraft (see page 167 of appendix III). The greater distance from the coast outweighed the advantages of dual runways (i.e., immediate dispersal due to aircraft taking off in opposite directions) making target 2 the most attractive, although targets 1 and 3 also drew a substantial number of aircraft. In this case, many aircraft would be left on the base when SLBMs arrive, as is clear from page 91 of appendix III which shows a zero inner annulus radius when weapons arrive, even when target 2 had only 15 aircraft. Consequently, it appears that too many aircraft are present to justify the assumption of uniform aircraft distribution, and the kills on target 2 could be underestimated by QUANTO.

APPENDIX III

QUANTO'S OUTPUT

```
CASE      1           MODE     3           DATE  06/15/73            TIME  11.41.33.
NEW PROBLEM WITH    4 TARGETS.    3 SUB LOCATIONS.    1 TYPES OF AIRCRAFT. AND    1 TYPES OF MISSILES.

TARGET LOCATIONS (DEGREES)    RUNWAYS  CENTROID  AIRCRAFT
                                        DISTANCE      TYPE AND NUMBER

     1   38.2500  103.2500          2    0.0000         1     15.0000

                             TAKE-OFF SEQUENCE BY TYPE      1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

     2   48.2500   97.5000          1    5.5000         1     15.0000

                             TAKE-OFF SEQUENCE BY TYPE      1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

     3   42.8500   91.4000          2    0.0000         1     15.0000

                             TAKE-OFF SEQUENCE BY TYPE      1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

     4   35.7000   85.9000          1    5.5000         1      5.0000

                             TAKE-OFF SEQUENCE BY TYPE      1 1 1 1 1

 AIRCRAFT  RELATIVE    BRAKE
     TYPE     VALUE  RELEASE       PSI       CAL
        1    1.0000   4.5000    1.5000        60

AIRCRAFT TAKE-OFF INTERVALS
       RUNWAYS     TYPE1     TYPE2   MINUTES
             1         1         1     .1667
             2         1         1     .0833

 SUB LOCATIONS (DEGREES)       SUBS   MISSILES AND TYPE
   1   36.7500   74.0000          1           2       1
   2   28.0500   93.4500          2           2       1
   3   45.6500  126.0500          2           2       1

NUMBER OF SUBS OF TYPE    1 =    5

  MISSILE    LAUNCH          RELIABILITIES                  MIN       MAX
     TYPE  INTERVAL     LAUNCH  FLIGHT WARHEAD            RANGE     RANGE      YIELD
        1     .2500      .9000   .9500   .9500          300.000  2000.000   1500.000
```

MISSILE TYPE	TIME	RANGE
1	4.3750	310.0000
	5.3190	520.0000
	6.4300	810.0000
	7.5417	1130.0000
	8.6528	1475.0000
	9.7639	1765.0000
	10.8750	2050.0000
	11.9861	2310.0000
	13.0972	2530.0000
	14.2083	2740.0000
	15.3195	2930.0000
	16.4305	3110.0000
	17.5417	3260.0000
	17.9861	3290.0000

SUBROUTINE PROCESS. CASE 1. MODE 3

AIRCRAFT TYPE 1 OF 1 TYPE(S)

VULNERABILITY CRITERIA FOR AIRCRAFT TYPE 1

1.50 PSI
60 CAL/CM**2

DATA INPUT FOR AIRCRAFT TYPE 1

CARD NUMBER	GROUND RANGE	FLIGHT TIME	AIRCRAFT ALTITUDE	VELOCITY OF SOUND	MACH NUMBER	LEVEL-OFF ACCELERATION
1	0.	0.	0.	1.11643653E+03	-0.000	-0.000
2	2.00000000E+03	2.00000000E+01	9.00000000E+00	1.11640205E+03	.141	-0.000
3	3.90000000E+03	3.00000000E+01	3.00000000E+01	1.11632158E+03	.141	-0.000
4	6.20000000E+03	4.00000000E+01	9.20000000E+01	1.11608398E+03	.195	-0.000
5	9.50000000E+03	5.00000000E+01	3.60000000E+02	1.11505635E+03	.254	-0.000
6	1.34000000E+04	6.00000000E+01	5.00000000E+02	1.11451915E+03	.315	-0.000
7	1.56000000E+04	6.60000000E+01	5.00010000E+02	1.11451912E+03	.355	-0.000
8	1.73000000E+04	6.80000000E+01	5.00020000E+02	1.11451908E+03	.367	-0.000
9	1.79000000E+04	6.90000000E+01	5.00030000E+02	1.11451904E+03	.373	-0.000
10	1.82000000E+04	7.10000000E+01	5.00040000E+02	1.11451900E+03	.386	-0.000
11	2.27500000E+04	7.80000000E+01	5.00050000E+02	1.11451896E+03	.435	-0.000
12	2.68000000E+04	8.60000000E+01	5.00060000E+02	1.11451892E+03	.462	-0.000
13	3.30000000E+04	9.40000000E+01	5.00070000E+02	1.11451889E+03	.564	-0.000
14	3.70000000E+04	1.01000000E+02	5.00080000E+02	1.11451885E+03	.596	-0.000
15	4.40000000E+04	1.09000000E+02	5.00090000E+02	1.11451881E+03	.638	-0.000
16	4.97500000E+04	1.17000000E+02	5.00100000E+02	1.11451977E+03	.662	-0.000
17	5.59000000E+04	1.25000000E+02	5.00110000E+02	1.11451873E+03	.686	-0.000
18	6.10000000E+04	1.32000000E+02	1.42500000E+03	1.11096328E+03	.704	-0.000
19	6.80000000E+04	1.40000000E+02	3.96000000E+03	1.10119825E+03	.727	-0.000
20	7.55000000E+04	1.48000000E+02	5.00000000E+03	1.09711197E+03	.731	6.200

INITIAL ALTITUDE (IN CLIMBING) OF MAXIMUM MACH

7600 FEET

DATA COMPUTED FOR AIRCRAFT TYPE 1 WITH RESPECT TO A TERMINAL ALTITUDE OF 5000 FEET

VELOCITY OF SOUND

1097.11 FEET/SECOND

ACCELERATION COMPONENT

6.200 FEET/SECOND/SECOND

MACH NUMBERS

INITIAL .731
TERMINAL .849

VELOCITY

INITIAL 802.2 FEET/SECOND
TERMINAL 931.4 FEET/SECOND

GROUND RANGE (FEET)	FLIGHT TIME (SEC)	GROUND RANGE (NM)	FLIGHT TIME (MIN)
0.	0.	0.	0.
2.00000000E+03	2.00000000E+01	3.28947368E-01	3.33333333E-01
3.90000000E+03	3.00000000E+01	6.41447368E-01	5.00000000E-01
6.20000000E+03	4.00000000E+01	1.01973684E+00	6.66666667E-01
9.50000000E+03	5.00000000E+01	1.56250000E+00	8.33333333E-01
1.34000000E+04	6.00000000E+01	2.20394737E+00	1.00000000E+00
1.56000000E+04	6.60000000E+01	2.56578947E+00	1.10000000E+00
1.73000000E+04	6.80000000E+01	2.84539474E+00	1.13333333E+00
1.79000000E+04	6.90000000E+01	2.94407895E+00	1.15000000E+00
1.82000000E+04	7.10000000E+01	2.99342105E+00	1.18333333E+00
2.27500000E+04	7.80000000E+01	3.74177632E+00	1.30000000E+00
2.68000000E+04	8.60000000E+01	4.40789474E+00	1.43333333E+00
3.30000000E+04	9.40000000E+01	5.42763158E+00	1.56666667E+00
3.70000000E+04	1.01000000E+02	6.08552632E+00	1.68333333E+00
4.40000000E+04	1.09000000E+02	7.23684211E+00	1.81666667E+00
4.97500000E+04	1.17000000E+02	8.18256579E+00	1.95000000E+00
5.59000000E+04	1.25000000E+02	9.19407895E+00	2.08333333E+00
6.10000000E+04	1.32000000E+02	1.00328947E+01	2.20000000E+00
6.80000000E+04	1.40000000E+02	1.11842105E+01	2.33333333E+00
7.55000000E+04	1.48000000E+02	1.24177632E+01	2.46666667E+00
7.71856835E+04	1.50084513E+02	1.26950137E+01	2.50140855E+00
7.88983071E+04	1.52169025E+02	1.29766952E+01	2.53615042E+00
8.06378710E+04	1.54253538E+02	1.32628077E+01	2.57089230E+00

8.24043750E+04	1.56338051E+02	1.35533512E+01	2.60563418E 00
8.41778192E+04	1.58422564E+02	1.38483255E+01	2.64037606E+00
8.60182037E+04	1.60507076E+02	1.41477309E+01	2.67511794E+00
8.78655283E+04	1.62591589E+02	1.44515672E+01	2.70985982E+00
8.97397932E+04	1.64676102E+02	1.47598344E+01	2.74460170E+00
9.16409982E+04	1.66760615E+02	1.50725326E+01	2.77934358E+00
9.35691435E+04	1.68845127E+02	1.53896618E+01	2.81408546E+00
1.44456027E+05	2.28845127E+02	2.45815834E+01	3.81408546E+00
2.05342911E+05	2.88845127E+02	3.37735050E+01	4.81408546E+00
2.61229794E+05	3.48845127E+02	4.29654267E+01	5.81408546E+00
3.17116678E+05	4.08845127E+02	5.21573483E+01	6.81408546E+00
3.73003561E+05	4.68845127E+02	6.13492699E+01	7.81408546E+00
4.28890445E+05	5.28 5127E+02	7.05411915E+01	8.81408546E+00
4.84777328E+05	5.88845127E+02	7.97331132E+01	9.81408546E+00
5.40664211E+05	6.48845127E+02	8.89250348E+01	1.08140855E+01
5.96551095E+05	7.08845127E+02	9.81169564E+01	1.18140855E+01
6.52437979E+05	7.68845127E+02	1.07308878E+02	1.28140855E+01
7.08324862E+05	8.28845127E+02	1.16500830E+02	1.38140855E+01
7.64211746E+05	8.88845127E+02	1.25692721E+02	1.48140855E+01
8.20098629E+05	9.48845127E+02	1.34884643E+02	1.58140855E+01
8.75985513E+05	1.00884513E+03	1.44076565E+02	1.68140855E+01
9.31872396E+05	1.06884513E+03	1.53268486E+02	1.78140855E+01
9.87759280E+05	1.12884513E+03	1.62460408E+02	1.88140855E+01
1.04364616E+06	1.18884513E+03	1.71652330E+02	1.98140855E+01
1.09953305E+06	1.24884513E+03	1.80844251E+02	2.08140855E+01
1.15541993E+06	1.30884513E+03	1.90036173E+02	2.18140855E+01
1.21130681E+06	1.36884513E+03	1.99228094E+02	2.28140855E+01
1.26719370E+06	1.42884513E+03	2.08420016E+02	2.38140855E+01
1.32308058E+06	1.48884513E+03	2.17611938E+02	2.48140855E+01
1.37896746E+06	1.54884513E+03	2.26803859E+02	2.58140855E+01
1.43485435E+06	1.60884513E+03	2.35995781E+02	2.68140855E+01
1.49074123E+06	1.66884513E+03	2.45187703E+02	2.78140855E+01
1.54662812E+06	1.72884513E+03	2.54379624E+02	2.88140855E+01
1.60251500E+06	1.78884513E+03	2.63571546E+02	2.98140855E+01
1.65840188E+06	1.84884513E+03	2.72763467E+02	3.08140855E+01
1.71428877E+06	1.90884513E+03	2.81955389E+02	3.18140855E+01
1.77017565E+06	1.96884513E+03	2.91147311E+02	3.28140855E+01
1.82606253E+06	2.02884513E+03	3.00339232E+02	3.38140855E+01
1.88194942E+06	2.08884513E+03	3.09531154E+02	3.48140855E+01
1.93783630E+06	2.14884513E+03	3.18723076E+02	3.58140855E+01
1.99372318E+06	2.20884513E+03	3.27914997E+02	3.68140855E+01
2.04961007E+06	2.26884513E+03	3.37106919E+02	3.78140855E+01
2.10549695E+06	2.32884513E+03	3.46298841E+02	3.88140855E+01
2.16138383E+06	2.38884513E+03	3.55490762E+02	3.98140855E+01
2.21727072E+06	2.44884513E+03	3.64682684E+02	4.08140855E+01
2.27315760E+06	2.50884513E+03	3.73874605E+02	4.18140855E+01
2.32904448E+06	2.56884513E+03	3.83066527E+02	4.28140855E+01
2.38493137E+06	2.62884513E+03	3.92258449E+02	4.38140855E+01
2.44081825E+06	2.68884513E+03	4.01450370E+02	4.48140855E+01
2.49670514E+06	2.74884513E+03	4.10642292E+02	4.58140855E+01

2.55259202E+06	2.80884513E+03	4.19834214E+02	4.68140855E+01
2.60847890E+06	2.86884513E+03	4.29026135E+02	4.78140855E+01
2.66436579E+06	2.92884513E+03	4.38218057E+02	4.88140855E+01
2.72025267E+06	2.98884513E+03	4.47409978E+02	4.98140855E+01
2.77613955E+06	3.04884513E+03	4.56601900E+02	5.08140855E+01
2.83202644E+06	3.10884513E+03	4.65793822E+02	5.18140855E+01
2.88791332E+06	3.16884513E+03	4.74985743E+02	5.28140855E+01

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MISSILE TYPE  1  OF  1  TYPE(S) AGAINST AIRCRAFT TYPE  1

    YIELD OF MISSILE TYPE  1

        1500  KT


SUBROUTINE SABERCM

     DATA INPUT TO SABERCM
          1.50  PSI      BLAST OVERPRESSURE
          1500  KT       YIELD
             0  FEET     TERRAIN HEIGHT
          2500  FEET     BURST HEIGHT
          5000  FEET     AIRCRAFT ALTITUDE

SABERCM OUTPUT FOLLOWS
                                        RANGE SOLUTION

W    15.00000E+02   DELP  15.00000E-01   HZ    50.00000E+02   HG     0.             HB    25.00000E+02

                    SR    35.62578E+00   RBAR  25.81295E-01   SDELP 17.31766E-01    FR    16.00000E-01

                    HORF  35.53796E+03   HORN  58.28225E-01

                    TSA   25.83402E+00   SFV   11.53339E+02   Q     64.58255E-03    PMV   91.44019E+00

                    POD   22.24540E-04   RHOZ  20.48172E-04   PUOOP 47.76316E-01    PDMV  57.04155E-01

                    PDOD  46.78741E-01   SSZ   10.97104E+02   P     27.02741E-01    ALFA  56.68854E-02

       YIELD CORRECTION FACTOR IS EQUAL TO ONE
     A SUMMARY OF DATA OUTPUT FROM SABERCM USED BY SUBROUTINE DETAREA IN COMPUTING LETHAL AREA
          LETHAL OVERPRESSURE RADIUS =   5.84505857E+00  NAUTICAL MILES
                                     =   3.55379561E+04  FEET
                TIME OF SHOCK ARRIVAL =   4.30567060E-01  MINUTES
                                     =   2.58340236E+01  SECONDS
```

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SUBROUTINE SNAPTCM

     DATA INPUT TO SNAPTCM
              60  CAL/CM**2  THERMAL ENERGY
            1500  KT         YIELD
               0  FEET       TERRAIN HEIGHT
            2500  FEET       BURST HEIGHT
            5000  FEET       AIRCRAFT ALTITUDE
           10000  FEET       HAZE LAYER HEIGHT
             5.0  MM HG      WATER VAPOR PRESSURE
            10.0  MILES      VISIBILITY(U.S., STATUTE MILES)
             .30  (ALBEDO)   GROUND REFLECTANCE
            1.00  (BETIND)   SHOULD BE 1.0

SNAPTC  OUTPUT FOLLOWS

PANEL DATA                                                  RECEIVER PARAMETERS

CRAFT  = 0.                  WTL   = 0.                     FTSEC  = 0.                  MAXALT = 0.
ALPHAL = 0.                  RTUL  = 0.                     ISTHR  = 2.6281362E+04       TILT   = 0.
TMPL   = 0.                  XLEL  = 0.                     ISTALT = 5.0000000E+03       BETAID = 1.0000000E+00
                                                            DALT   = 0.                  CAL    = 6.0000000E+01

SOURCE PARAMETERS                                           ATMOSPHERIC PARAMETERS

YIELD  = 1.5000000E+03                                      ATM    = 0.                  VISBLE = 1.0000000E+01
BURST  = 2.5000000E+03                                      DAY    = 0.                  VAPOR  = 5.0000000E+00
TARGET = 0.                                                 ALBEDO = 3.0000000E-01       HAZE   = 1.0000000E+04
TH EFF = 4.3224397E-01
DBURST = 0.
MBURST = 0.

RECEIVER ALTITUDE           (FT) AZ = 5.00000E+03    BURST ALTITUDE  (FT)  HBL = 2.50000E+03
                            (KM) AZ = 1.52393E+00                    (KM)  HBL = 7.61963E-01
CALCULATED HORIZONTAL RANGE (FT) SZ = 2.28061E+04    ANGLE BETWEEN LOCAL HORIZONTAL
                            (KM) SZ = 6.95096E+00    AND CRITICAL PANEL (RADIANS) BETA = 1.46085E+00
                            (NM) SZ = 3.74020E+00

UNATTENUATED ENERGY IN LOWER PHASE (CAL/CM**2)  CL = 7.97458E+01
UNATTENUATED ENERGY IN UPPER PHASE (CAL/CM**2)  CU = 1.05826E+01

ATTENUATED ENERGY IN LOWER PHASE (CAL/CM**2)             QEL = 5.18066E+01   QELD = 5.08345E+01   QELR = 9.72130E-01
ATTENUATED ENERGY IN UPPER PHASE (CAL/CM**2)             QEU = 8.18799E+00   QEUD = 6.83732E+00   QEUR = 1.35067E+00

TOTAL FREE FIELD ENERGY ATCRITICAL PANEL (CAL/CM**2)     QA = 6.00000E+01
TOTAL ITERATED ENERGY AT CRITICAL PANEL  (CAL/CM**2)     QE = 5.99946E+01

      A SUMMARY OF DATA OUTPUT FROM SNAPTCM USED BY SUBROUTINE DETAREA IN COMPUTING LETHAL AREA
          LETHAL THERMAL RADIUS =   3.75100380E+00  NAUTICAL MILES
                                =   2.28061092E+04  FEET
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SUBROUTINE DETAREA NUCLEAR LOOKUP - (D+DSPT) = (  0.00 +   0.00) =   0.00 NM, WHERE DISMIN =  12.42 NM
         MISSILE TYPE  1  OF  1  TYPE(S) AGAINST AIRCRAFT TYPE  1

            YIELD OF MISSILE TYPE  1
                 1500  KT




          SUBROUTINE SABERCM

               DATA INPUT TO SABERCM
                    1.50  PSI      BLAST OVERPRESSURE
                    1500  KT       YIELD
                       0  FEET     TERRAIN HEIGHT
                    2500  FEET     BURST HEIGHT
                       0  FEET     AIRCRAFT ALTITUDE

          SABERCM OUTPUT FOLLOWS
                                           RANGE SOLUTION


   W    15.00000E+02   DELP  15.00000E-01   HZ    0.              HG    0.             HB    25.00000E+02

                       SR    36.56196E+00   RBAR  27.94038E-01   SDELP 15.60227E-01    FR    16.00000E-01

                       HORF  38.48083E+03   HORN  63.10856E-01

                       TSA   26.32797E+00   SFV   11.64271E+02   Q     53.89394E-03    PMV   78.05405E+00

                       POD   25.47701E-04   RHOZ  23.76900E-04   PDOOP 45.27825E-01    PDNV  53.41786E-01

                       PDOD  43.98499E-01   SSZ   11.16457E+02   R     27.02741E-01    ALFA  56.79006E-02

                  YIELD CORRECTION FACTOR IS EQUAL TO ONE
                A SUMMARY OF DATA OUTPUT FROM SABERCM USED BY SUBROUTINE DETAREA IN COMPUTING LETHAL AREA
                    LETHAL OVERPRESSURE RADIUS =   6.32908416E+00  NAUTICAL MILES
                                               =   3.84808317E+04  FEET
                          TIME OF SHOCK ARRIVAL =  4.38799471E-01  MINUTES
                                               =   2.63279682E+01  SECONDS
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```
SUBROUTINE SNAPTCM

     DATA INPUT TO SNAPTCM
              60   CAL/CM**2   THERMAL ENERGY
            1500   KT          YIELD
               0   FEET        TERRAIN HEIGHT
            2500   FEET        BURST HEIGHT
               0   FEET        AIRCRAFT ALTITUDE
           10000   FEET        HAZE LAYER HEIGHT
             5.0   MM HG       WATER VAPOR PRESSURE
            10.0   MILES       VISIBILITY(U.S., STATUTE MILES)
             .30   (ALBEDO)    GROUND REFLECTANCE
            1.00   (BETIND)    SHOULD BE 1.0

     SNAPTCM OUTPUT FOLLOWS

PANEL DATA                                                     RECEIVER PARAMETERS

CRAFT  = 0.                 WTL  = 0.                          FTSEC  = 0.                  MAXALT = 0.
ALPHAL = 0.                 RTUL = 0.                          ISTHR  = 2.6281362E+04       TILT   = 0.
THPL   = 0.                 XLEL = 0.                          ISTALT = 0.                  BETAID = 1.0000000E+00
                                                               DALT   = 0.                  CAL    = 6.0000000E+01

SOURCE PARAMETERS
                                                               ATMOSPHERIC PARAMETERS
YIELD  = 1.5000000E+03
BURST  = 2.5000000E+03                                         ATM    = 0.                  VISBLE = 1.0000000E+01
TARGET = 0.                                                    DAY    = 0.                  VAPOR  = 5.0000000E+00
TH EFF = 4.3224397E-01                                         ALBEDO = 3.0000000E-01       HAZE   = 1.0000000E+04
DBURST = 0.
MBURST = 0.


RECEIVER ALTITUDE              (FT) AZ = 0.            BURST ALTITUDE  (FT)  HBL = 2.50000E+03
                               (KM) AZ = 0.                            (KM)  HBL = 7.61963E-01
CALCULATED HORIZONTAL RANGE    (FT) SZ = 2.14475E+04   ANGLE BETWEEN LOCAL HORIZONTAL
                               (KM) SZ = 6.53689E+00   AND CRITICAL PANEL (RADIANS) BETA = 1.68073E+00
                               (NM) SZ = 3.51740E+00

UNATTENUATED ENERGY IN LOWER PHASE (CAL/CM**2)   CL = 8.21627E+01
UNATTENUATED ENERGY IN UPPER PHASE (CAL/CM**2)   CU = 9.97587E+00

ATTENUATED ENERGY IN LOWER PHASE (CAL/CM**2)              QEL = 5.20776E+01    QELD = 5.10891E+01    QELR = 9.88562E-01
ATTENUATED ENERGY IN UPPER PHASE (CAL/CM**2)              QEU = 7.91678E+00    QEUD = 6.23198E+00    QEUR = 1.68481E+00

TOTAL FREE FIELD ENERGY ATCRITICAL PANEL (CAL/CM**2)      QA = 6.00000E+01
TOTAL ITERATED ENERGY AT CRITICAL PANEL  (CAL/CM**2)      QE = 5.99944E+01

        A SUMMARY OF DATA OUTPUT FROM SNAPTCM USED BY SUBROUTINE DETAREA IN COMPUTING LETHAL AREA
           LETHAL THERMAL RADIUS =    3.52755671E+00  NAUTICAL MILES
                                 =    2.14475448E+04  FEET
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SUBROUTINE DETAREA NUCLEAR LOOKUP - (O+DSPT) = ( 1.00 +   0.00) =   1.00 NM, WHERE DISMIN =  12.42 NM

        MISSILE TYPE  1  OF  1  TYPE(S) AGAINST AIRCRAFT TYPE  1

           YIELD OF MISSILE TYPE  1
                1500  KT



           SUBROUTINE SABERCM

                DATA INPUT TO SABERCM
                      1.50  PSI      BLAST OVERPRESSURE
                      1500  KT       YIELD
                         0  FEET     TERRAIN HEIGHT
                      2500  FEET     BURST HEIGHT
                        87  FEET     AIRCRAFT ALTITUDE

            SABERCM OUTPUT FOLLOWS
                                                      RANGE SOLUTION


 W     15.00000E+02   DELP  15.00000E-01   HZ    86.93474E+00   HG     0.              HB   25.00000E+02

                      SR    38.50907E+00   RBAP  27.90206E-01   SDELP 15.63016E-01     FR   16.00000E-01

                      HORF  38.43339E+03   HORN  63.03074E-01

                      TSA   26.31854E+00   SFV   11.64071E+02   Q      54.06113E-03    PMV  78.26641E+00

                      POD   25.41754E-04   RH07  23.70859E-04   PDOCP 45.31965E-01     PDMV 53.47822E-01

                      PODD  44.03166E-01   SS7   11.16124E+02   R      27.02741E-01    ALFA 56.78879E-02

               YIELD CORRECTION FACTOR IS EQUAL TO ONE
             A SUMMARY OF DATA OUTPUT FROM SABERCM USED BY SUBROUTINE DETAREA IN COMPUTING LETHAL AREA
                 LETHAL OVERPRESSURE RADIUS =   6.32128175E+00  NAUTICAL MILES
                                            =   3.84333930E+04  FEET
                      TIME OF SHOCK ARRIVAL =   4.38642289E-01  MINUTES
                                            =   2.63185374E+01  SECONDS
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SUBROUTINE SNAPTCM

     DATA INPUT TO SNAPTCM
                60  CAL/CM**2   THERMAL ENERGY
              1500  KT          YIELD
                 0  FEET        TERRAIN HEIGHT
              2500  FEET        BURST HEIGHT
                67  FEET        AIRCRAFT ALTITUDE
             10000  FEET        HAZE LAYER HEIGHT
               5.0  MM HG       WATER VAPOR PRESSURE
              10.0  MILES       VISIBILITY(U.S.. STATUTE MILES)
               .30  (ALBEDO)    GROUND REFLECTANCE
              1.00  (BETIND)    SHOULD BE 1.0

     SNAPTCM OUTPUT FOLLOWS

PANEL DATA                                          RECEIVER PARAMETERS

CRAFT  = 0.          WTL   = 0.                     FTSFC  = 0.                MAXALT = 0.
ALPHAL = 0.          RTUL  = 0.                     ISTHR  = 2.6289487E+04     TILT   = 0.
TMPL   = 0.          XLEL  = 0.                     ISTALT = 8.6934736E+01     BETAID = 1.0000000E+00
                                                    DALT   = 0.                CAL    = 6.0000000E+01

SOURCE PARAMETERS                                   ATMOSPHERIC PARAMETERS

YIELD  = 1.5000000E+03                              ATM    = 0.                VISBLE = 1.0000000E+01
BURST  = 2.5000000E+03                              DAY    = 0.                VAPOR  = 5.0000000E+00
TARGET = 0.                                         ALBEDO = 3.0000000E-01     HAZE   = 1.0000000E+04
TH EFF = 4.3224397E-01
DBURST = 0.
MBURST = 0.

RECEIVER ALTITUDE              (FT) AZ = 8.69347E+01    BURST ALTITUDE  (FT)  HBL = 2.50000E+03
                               (KM) AZ = 2.64964E-02                    (KM)  HBL = 7.61963E-01
CALCULATED HORIZONTAL RANGE    (FT) SZ = 2.14800E+04    ANGLE BETWEEN LOCAL HORIZONTAL
                               (KM) SZ = 6.54676E+00    AND CRITICAL PANEL (RADIANS) BETA = 1.67[illegible]E+00
                               (NM) SZ = 3.52272E+00

UNATTENUATED ENERGY IN LOWER PHASE (CAL/CM**2)  CL = 8.21441E+01
UNATTENUATED ENERGY IN UPPER PHASE (CAL/CM**2)  CU = 9.99064E+00

ATTENUATED ENERGY IN LOWER PHASE (CAL/CM**2)          QEL = 5.20842E+01   QELD = 5.11035E+01   QELR = 9.80644E-01
ATTENUATED ENERGY IN UPPER PHASE (CAL/CM**2)          QEU = 7.91033E+00   QEUD = 6.24514E+00   QEUR = 1.66520E+00

TOTAL FREE FIELD ENERGY ATCRITICAL PANEL (CAL/CM**2)   QA = 6.00000E+01
TOTAL ITERATED ENERGY AT CRITICAL PANEL  (CAL/CM**2)   QE = 5.99945E+01

     A SUMMARY OF DATA OUTPUT FROM SNAPTCM USED BY SUBROUTINE DETAREA IN COMPUTING LETHAL AREA
          LETHAL THERMAL RADIUS =   3.53289196E+00  NAUTICAL MILES
                                =   2.14799831E+04  FEET
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FOR DISTANCE OF 0.00 NM TO CENTROID FROM START OF TAKE-OFF ROLL

AIRCRAFT TYPE 1 VERSUS MISSILE TYPE 1

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WHEN DETONATION IS   0.00 NM FROM CENTROID, LETHAL AREA =  39.82 SQUARE NM AND EXTENDS 3.56 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   1.00 NM FROM CENTROID, LETHAL AREA =  40.64 SQUARE NM AND EXTENDS 3.53 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   2.00 NM FROM CENTROID, LETHAL AREA =  43.24 SQUARE NM AND EXTENDS 3.56 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   3.00 NM FROM CENTROID, LETHAL AREA =  46.66 SQUARE NM AND EXTENDS 3.56 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   4.00 NM FROM CENTROID, LETHAL AREA =  50.33 SQUARE NM AND EXTENDS 3.56 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   5.00 NM FROM CENTROID, LETHAL AREA =  54.78 SQUARE NM AND EXTENDS 3.56 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   6.00 NM FROM CENTROID, LETHAL AREA =  60.22 SQUARE NM AND EXTENDS 3.56 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   7.00 NM FROM CENTROID, LETHAL AREA =  65.51 SQUARE NM AND EXTENDS 3.56 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   8.00 NM FROM CENTROID, LETHAL AREA =  70.32 SQUARE NM AND EXTENDS 3.56 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   9.00 NM FROM CENTROID, LETHAL AREA =  74.75 SQUARE NM AND EXTENDS 3.55 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  10.00 NM FROM CENTROID, LETHAL AREA =  78.30 SQUARE NM AND EXTENDS 3.60 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  15.00 NM FROM CENTROID, LETHAL AREA =  88.49 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  20.00 NM FROM CENTROID, LETHAL AREA =  97.02 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  25.00 NM FROM CENTROID, LETHAL AREA = 102.40 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  30.00 NM FROM CENTROID, LETHAL AREA = 105.23 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  35.00 NM FROM CENTROID, LETHAL AREA = 107.08 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  40.00 NM FROM CENTROID, LETHAL AREA = 108.52 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  45.00 NM FROM CENTROID, LETHAL AREA = 109.63 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  50.00 NM FROM CENTROID, LETHAL AREA = 110.52 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  55.00 NM FROM CENTROID, LETHAL AREA = 111.25 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  60.00 NM FROM CENTROID, LETHAL AREA = 111.85 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  65.00 NM FROM CENTROID, LETHAL AREA = 112.37 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  70.00 NM FROM CENTROID, LETHAL AREA = 112.81 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  75.00 NM FROM CENTROID, LETHAL AREA = 113.19 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  80.00 NM FROM CENTROID, LETHAL AREA = 113.52 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  85.00 NM FROM CENTROID, LETHAL AREA = 113.81 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  90.00 NM FROM CENTROID, LETHAL AREA = 114.07 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  95.00 NM FROM CENTROID, LETHAL AREA = 114.30 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS 100.00 NM FROM CENTROID, LETHAL AREA = 114.51 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS 105.00 NM FROM CENTROID, LETHAL AREA = 114.70 SQUARE NM AND EXTENDS 3.75 NM FARTHER FROM CENTROID.
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TARGET     1 BRAKE RELEASE TIMES
AIRCRAFT TYPE =     1 STARTS AT    4.50 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    4.58 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    4.67 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    4.75 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    4.83 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    4.92 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.00 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.08 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.17 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.25 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.33 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.42 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.50 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.58 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.67 MINUTES.

DISTANCE TO TARGET    1 FROM SUB LOCATION    1 IS    1391.25 NM. FLIGHT TIME (MISSILE TYPE  1) =  8.37354084E+00 MIN.
WEAPON     1 ARRIVES ON TARGET    1 WHEN FIRST AIRCRAFT IS      25.13 NAUTICAL MILES BEYOND CENTROID.
ANNULUS PK =  .0721, CIRCULAR PK =  .0471.  WEAPON    1 VS. AIRCRAFT TYPE    1.  ANNULAR RADII ARE   25.1281 AND    14.4294

DISTANCE TO TARGET    1 FROM SUB LOCATION    2 IS     785.40 NM. FLIGHT TIME (MISSILE TYPE  1) =  6.34162586E+00 MIN.
WEAPON     3 ARRIVES ON TARGET    1 WHEN FIRST AIRCRAFT IS       7.42 NAUTICAL MILES BEYOND CENTROID.
ANNULUS PK =  .2720, CIRCULAR PK =  .2878.  WEAPON    3 VS. AIRCRAFT TYPE    1.  ANNULAR RADII ARE    7.4213 AND     1.0447
ANNULUS PK =  .2113. CIRCULAR PK =  .2176.  WEAPON    4 VS. AIRCRAFT TYPE    1.  ANNULAR RADII ARE    9.2527 AND     1.9156

DISTANCE TO TARGET    1 FROM SUB LOCATION    3 IS    1106.51 NM. FLIGHT TIME (MISSILE TYPE  1) =  7.46331120E+00 MIN.
WEAPON     5 ARRIVES ON TARGET    1 WHEN FIRST AIRCRAFT IS      16.75 NAUTICAL MILES BEYOND CENTROID.
ANNULUS PK =  .1130, CIRCULAR PK =  .0943,  WEAPON    5 VS. AIRCRAFT TYPE    1.  ANNULAR RADII ARE   16.7542 AND     7.0651
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SUBROUTINE DETAREA NUCLEAR LOOKUP - (O+USPT) = (  0.00 +   5.50) =   5.50 NM, WHERE DISMIN =  12.42 NM

         MISSILE TYPE  1  OF  1  TYPE(S) AGAINST AIRCRAFT TYPE  1

            YIELD OF MISSILE TYPE  1
                 1500  KT




            SUBROUTINE SABERCM

                 DATA INPUT TO SABERCM
                       1.50  PSI      BLAST OVERPRESSURE
                       1500  KT       YIELD
                          0  FEET     TERRAIN HEIGHT
                       2500  FEET     BURST HEIGHT
                        500  FEET     AIRCRAFT ALTITUDE

            SABERCM OUTPUT FOLLOWS
                                                RANGE SOLUTION


   W     15.00000E+02    DELP  15.00000E-01    HZ     50.00711E+0:    HG     0.               HB    25.00000E+02

                         SR    38.25864E+00    RBAR   27.72061E-01    SDELP  15.76362E-01     FR    16.00000E-01

                         HORF  38.20633E+03    HORN   62.65838E-01

                         TSA   26.27376E+00    SFV    11.63122E+02    Q      54.86401E-03     PMV   79.28488E+00

                         POD   25.13658E-04    RHOZ   23.42313E-04    PDOOP  45.51721E-01     PDMV  53.76627E-01

                         PDOD  44.25440E-01    SSZ    11.14536E+02    P      27.02741E-01     ALFA  56.78251E-02

                YIELD CORRECTION FACTOR IS EQUAL TO ONE
               A SUMMARY OF DATA OUTPUT FROM SABERCM USED BY SUBROUTINE DETAREA IN COMPUTING LETHAL AREA
                   LETHAL OVERPRESSURE RADIUS =    6.28393601E+00  NAUTICAL MILES
                                              =    3.82063309E+04  FEET
                        TIME OF SHOCK ARRIVAL =    4.37895969E-01  MINUTES
                                              =    2.62737581E+01  SECONDS
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SUBROUTINE SNAPTCM

     DATA INPUT TO SNAPTCM
               60   CAL/CM**2     THERMAL ENERGY
             1500   KT            YIELD
                0   FEET          TERRAIN HEIGHT
             2500   FEET          BURST HEIGHT
              500   FEET          AIRCRAFT ALTITUDE
            10000   FEET          HAZE LAYER HEIGHT
              5.0   MM HG         WATER VAPOR PRESSURE
             10.0   MILES         VISIBILITY(U.S.. STATUTE MILES)
              .30   (ALBEDO)      GROUND REFLECTANCE
             1.00   (HFTIND)      SHOULD BE 1.0

          SNAPTCM OUTPUT FOLLOWS

PANEL DATA                                                     RECEIVER PARAMETERS

CRAFT  = 0.                   WTL    = 0.                      FTSEC  = 0.                  MAXALT = 0.
ALPHAL = 0.                   RTUL   = 0.                      ISTHR  = 2.6324139E+04       TILT   = 0.
TMPL   = 0.                   XLEL   = 0.                      ISTALT = 5.0007108E+02       BETAID = 1.0000000E+00
                                                               DALT   = 0.                  CAL    = 6.0000000E+01

SOURCE PARAMETERS                                              ATMOSPHERIC PARAMETERS

YIELD  = 1.5000000E+03                                         ATM    = 0.                  VISBLE = 1.0000000E+01
BURST  = 2.5000000E+03                                         DAY    = 0.                  VAPOR  = 5.0000000E+00
TARGET = 0.                                                    ALBEDO = 3.0000000E-01       HAZE   = 1.0000000E+04
TH EFF = 4.3224397E-01
DBURST = 0.
MBURST = 0.

RECEIVER ALTITUDE               (FT) AZ = 5.00071E+02    BURST ALTITUDE  (FT)  HBL = 2.50000E+03
                                (KM) AZ = 1.52414E-01                    (KM)  HBL = 7.61963E-01
CALCULATED HORIZONTAL RANGE     (FT) SZ = 2.16291E+04    ANGLE BETWEEN LOCAL HORIZONTAL
                                (KM) SZ = 6.59223E+00    AND CRITICAL PANEL (RADIANS) BETA = 1.65831E+00
                                (NM) SZ = 3.54718E+00

UNATTENUATED ENERGY IN LOWER PHASE (CAL/CM**2)   CL = 8.20459E+01
UNATTENUATED ENERGY IN UPPER PHASE (CAL/CM**2)   CU = 1.00591E+01

ATTENUATED ENERGY IN LOWER PHASE (CAL/CM**2)              QEL = 5.21085E+01    QELD = 5.11639E+01   QELR = 9.44629E-01
ATTENUATED ENERGY IN UPPER PHASE (CAL/CM**2)              QEU = 7.88635E+00    QEUD = 6.30641E+00   QEUR = 1.57993E+00

TOTAL FREE FIELD ENERGY ATCRITICAL PANEL (CAL/CM**2)      QA = 6.00000E+01
TOTAL ITERATED ENERGY AT CRITICAL PANEL  (CAL/CM**2)      QE = 5.99949E+01

        A SUMMARY OF DATA OUTPUT FROM SNAPTCM USED BY SUBROUTINE DETAREA IN COMPUTING LETHAL AREA
            LETHAL THERMAL RADIUS =   3.55742070E+00  NAUTICAL MILES
                                  =   2.16291179E+04  FEET
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SUBROUTINE DETAREA NUCLEAR LOOKUP - (O+DSPT) = (  1.00 +   5.50) =   6.50 NM, WHERE DISMIN =  12.42 NM

        MISSILE TYPE  1  OF  1  TYPE(S) AGAINST AIRCRAFT TYPE  1

           YIELD OF MISSILE TYPE  1
                1500  KT



          SUBROUTINE SABERCM

               DATA INPUT TO SABERCM
                    1.50  PSI      BLAST OVERPRESSURE
                    1500  KT       YIELD
                       0  FEET     TERRAIN HEIGHT
                    2500  FEET     BURST HEIGHT
                     500  FEET     AIRCRAFT ALTITUDE

          SABERCM OUTPUT FOLLOWS
                                                  RANGE SOLUTION


 W      15.00000E+02    DELP  15.00000E-01    HZ    50.60842E+01    HG     0.                HB    25.00000E+02

                        SR    38.25863E+00    RBAR  27.72061E-01    SDELP 15.76363E-01    FR    16.00000E-01

                        HORF  38.20632E+03    HORN  62.65837E-01

                        TSA   26.27376E+00    SFV   11.63122E+02    O     54.86404E-03    PMV   79.28491E+00

                        PUD   25.13657E-04    RHOZ  23.42312E-04    PDOOP 45.51722E-01    PDMV  53.76628E-01

                        PDOD  44.25441E-01    SSZ   11.14536E+02    R     27.02741E-01    ALFA  56.78251E-02

                         YIELD CORRECTION FACTOR IS EQUAL TO ONE
                       A SUMMARY OF DATA OUTPUT FROM SABERCM USED BY SUBROUTINE DETAREA IN COMPUTING LETHAL AREA
                            LETHAL OVERPRESSURE RADIUS =   6.28393482E+00  NAUTICAL MILES
                                                       =   3.82063237E+04  FEET
                                 TIME OF SHOCK ARRIVAL =   4.37895945E-01  MINUTES
                                                       =   2.62737567E+01  SECONDS
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SUBROUTINE SNAPTCM

     DATA INPUT TO SNAPTCM
              60  CAL/CM**2   THERMAL ENERGY
            1500  KT          YIELD
               0  FEET        TERRAIN HEIGHT
            2500  FEET        BURST HEIGHT
             500  FEET        AIRCRAFT ALTITUDE
           10000  FEET        HAZE LAYER HEIGHT
             5.0  MM HG       WATER VAPOR PRESSURE
            10.0  MILES       VISIBILITY(U.S., STATUTE MILES)
             .30  (ALBEDO)    GROUND REFLECTANCE
            1.00  (RETIND)    SHOULD BE 1.0

   SNAPTCM OUTPUT FOLLOWS

PANEL DATA                                                RECEIVER PARAMETERS

CRAFT  = 0.                    WTL    = 0.                FTSEC  = 0.                    MAXALT = 0.
ALPHAL = 0.                    RTUL   = 0.                ISTHR  = 2.6324140E+04         TILT   = 0.
TMPL   = 0.                    XLEL   = 0.                ISTALT = 5.0008416E+02         BETAID = 1.0000000E+00
                                                          DALT   = 0.                    CAL    = 6.0000000E+01

SOURCE PARAMETERS                                         ATMOSPHERIC PARAMETERS

YIELD  = 1.5000000E+03                                    ATM    = 0.                    VISBLE = 1.0000000E+01
BURST  = 2.5000000E+03                                    DAY    = 0.                    VAPOR  = 5.0000000E+00
TARGET = 0.                                               ALBEDO = 3.0000000E-01         HAZE   = 1.0000000E+04
TH EFF = 4.3224397E-01
DBURST = 0.
MBURST = 0.

RECEIVER ALTITUDE             (FT) AZ = 5.00084E+02    BURST ALTITUDE  (FT)  HBL = 2.50000E+03
                              (KM) AZ = 1.52418E-01                    (KM)  HBL = 7.61963E-01
CALCULATED HORIZONTAL RANGE   (FT) SZ = 2.16291E+04    ANGLE BETWEEN LOCAL HORIZONTAL
                              (KM) SZ = 6.59223E+00    AND CRITICAL PANEL (RADIANS) BETA = 1.65831E+00
                              (NM) SZ = 3.54718E+00

UNATTENUATED ENERGY IN LOWER PHASE (CAL/CM**2)  CL = 8.20459E+01
UNATTENUATED ENERGY IN UPPER PHASE (CAL/CM**2)  CU = 1.00591E+01

ATTENUATED ENERGY IN LOWER PHASE (CAL/CM**2)             QEL = 5.21085E+01   QELD = 5.11639E+01   QELR = 9.44627E-01
ATTENUATED ENERGY IN UPPER PHASE (CAL/CM**2)             QEU = 7.88635E+00   QEUD = 6.30641E+00   QEUR = 1.57993E+00

TOTAL FREE FIELD ENERGY ATCRITICAL PANEL (CAL/CM**2)     QA = 6.00000E+01
TOTAL ITERATED ENERGY AT CRITICAL PANEL  (CAL/CM**2)     QE = 5.99949E+01

       A SUMMARY OF DATA OUTPUT FROM SNAPTCM USED BY SUBROUTINE DETAREA IN COMPUTING LETHAL AREA
          LETHAL THERMAL RADIUS =   3.55742146E+00  NAUTICAL MILES
                                =   2.16291225E+04  FEET
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FOR DISTANCE OF  5.50 NM TO CENTROID FROM START OF TAKE-OFF ROLL

AIRCRAFT TYPE    1 VERSUS MISSILE TYPE    1

WHEN DETONATION IS   0.00 NM FROM CENTROID, LETHAL AREA =   39.76 SQUARE NM AND EXTENDS  3.56 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   1.00 NM FROM CENTROID, LETHAL AREA =   39.76 SQUARE NM AND EXTENDS  3.56 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   2.00 NM FROM CENTROID, LETHAL AREA =   39.76 SQUARE NM AND EXTENDS  3.56 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   3.00 NM FROM CENTROID, LETHAL AREA =   39.68 SQUARE NM AND EXTENDS  3.55 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   4.00 NM FROM CENTROID, LETHAL AREA =   44.34 SQUARE NM AND EXTENDS  3.57 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   5.00 NM FROM CENTROID, LETHAL AREA =   50.04 SQUARE NM AND EXTENDS  3.66 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   6.00 NM FROM CENTROID, LETHAL AREA =   55.64 SQUARE NM AND EXTENDS  3.74 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   7.00 NM FROM CENTROID, LETHAL AREA =   61.15 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   8.00 NM FROM CENTROID, LETHAL AREA =   66.66 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS   9.00 NM FROM CENTROID, LETHAL AREA =   71.75 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  10.00 NM FROM CENTROID, LETHAL AREA =   76.02 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  15.00 NM FROM CENTROID, LETHAL AREA =   90.43 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  20.00 NM FROM CENTROID, LETHAL AREA =   98.33 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  25.00 NM FROM CENTROID, LETHAL AREA =  102.52 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  30.00 NM FROM CENTROID, LETHAL AREA =  105.16 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  35.00 NM FROM CENTROID, LETHAL AREA =  107.08 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  40.00 NM FROM CENTROID, LETHAL AREA =  108.52 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  45.00 NM FROM CENTROID, LETHAL AREA =  109.62 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  50.00 NM FROM CENTROID, LETHAL AREA =  110.52 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  55.00 NM FROM CENTROID, LETHAL AREA =  111.25 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  60.00 NM FROM CENTROID, LETHAL AREA =  111.85 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  65.00 NM FROM CENTROID, LETHAL AREA =  112.37 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  70.00 NM FROM CENTROID, LETHAL AREA =  112.81 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  75.00 NM FROM CENTROID, LETHAL AREA =  113.19 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  80.00 NM FROM CENTROID, LETHAL AREA =  113.52 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  85.00 NM FROM CENTROID, LETHAL AREA =  113.81 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  90.00 NM FROM CENTROID, LETHAL AREA =  114.07 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS  95.00 NM FROM CENTROID, LETHAL AREA =  114.30 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS 100.00 NM FROM CENTROID, LETHAL AREA =  114.51 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
WHEN DETONATION IS 105.00 NM FROM CENTROID, LETHAL AREA =  114.70 SQUARE NM AND EXTENDS  3.75 NM FARTHER FROM CENTROID.
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TARGET     2 BRAKE RELEASE TIMES
AIRCRAFT TYPE =    1 STARTS AT   4.50 MINUTES.
AIRCRAFT TYPE =    1 STARTS AT   4.67 MINUTES.
AIRCRAFT TYPE =    1 STARTS AT   4.83 MINUTES.
AIRCRAFT TYPE =    1 STARTS AT   5.00 MINUTES.
AIRCRAFT TYPE =    1 STARTS AT   5.17 MINUTES.
AIRCRAFT TYPE =    1 STARTS AT   5.33 MINUTES.
AIRCRAFT TYPE =    1 STARTS AT   5.50 MINUTES.
AIRCRAFT TYPE =    1 STARTS AT   5.67 MINUTES.
AIRCRAFT TYPE =    1 STARTS AT   5.83 MINUTES.
AIRCRAFT TYPE =    1 STARTS AT   6.00 MINUTES.
AIRCRAFT TYPE =    1 STARTS AT   6.17 MINUTES.
AIRCRAFT TYPE =    1 STARTS AT   6.33 MINUTES.
AIRCRAFT TYPE =    1 STARTS AT   6.50 MINUTES.
AIRCRAFT TYPE =    1 STARTS AT   6.67 MINUTES.
AIRCRAFT TYPE =    1 STARTS AT   6.83 MINUTES.

DISTANCE TO TARGET   2 FROM SUB LOCATION   1 IS   1241.38 NM. FLIGHT TIME (MISSILE TYPE  1) =  7.89453438E+00 MIN.
WEAPON    1 ARRIVES ON TARGET    2 WHEN FIRST AIRCRAFT IS     15.21 NAUTICAL MILES BEYOND CENTROID.
ANNULUS PK =  .0887, CIRCULAR PK =  .1085,  WEAPON    1 VS. AIRCRAFT TYPE    1.  ANNULAR RADII ARE   15.2146 AND    0.0000
ANNULUS PK =  .0732, CIRCULAR PK =  .0873,  WEAPON    2 VS. AIRCRAFT TYPE    1.  ANNULAR RADII ARE   17.5181 AND    0.0000

DISTANCE TO TARGET   2 FROM SUB LOCATION   2 IS   1228.12 NM. FLIGHT TIME (MISSILE TYPE  1) =  7.85267974E+00 MIN.
WEAPON    3 ARRIVES ON TARGET    2 WHEN FIRST AIRCRAFT IS     14.83 NAUTICAL MILES BEYOND CENTROID.
ANNULUS PK =  .0919, CIRCULAR PK =  .1127,  WEAPON    3 VS. AIRCRAFT TYPE    1.  ANNULAR RADII ARE   14.8294 AND    0.0000
ANNULUS PK =  .0755, CIRCULAR PK =  .0904,  WEAPON    4 VS. AIRCRAFT TYPE    1.  ANNULAR RADII ARE   17.1322 AND    0.0000

DISTANCE TO TARGET   2 FROM SUB LOCATION   3 IS   1174.33 NM. FLIGHT TIME (MISSILE TYPE  1) =  7.68270606E+00 MIN.
WEAPON    5 ARRIVES ON TARGET    2 WHEN FIRST AIRCRAFT IS     13.27 NAUTICAL MILES BEYOND CENTROID.
ANNULUS PK =  .1070, CIRCULAR PK =  .1329,  WEAPON    5 VS. AIRCRAFT TYPE    1.  ANNULAR RADII ARE   13.2665 AND    0.0000
ANNULUS PK =  .0860, CIRCULAR PK =  .1048,  WEAPON    6 VS. AIRCRAFT TYPE    1.  ANNULAR RADII ARE   15.5661 AND    0.0000
```

```
TARGET     3 BRAKE RELEASE TIMES
AIRCRAFT TYPE =     1 STARTS AT    4.50 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    4.58 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    4.67 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    4.75 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    4.83 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    4.92 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.00 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.08 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.17 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.25 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.33 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.42 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.50 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.58 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.67 MINUTES.

DISTANCE TO TARGET    3 FROM SUB LOCATION    1 IS    880.57 NM. FLIGHT TIME (MISSILE TYPE  1) =  6.68409605E+00 MIN.
WEAPON     1 ARRIVES ON TARGET     3 WHEN FIRST AIRCRAFT IS      9.91 NAUTICAL MILES BEYOND CENTROID.
ANNULUS PK =  .1967, CIRCULAR PK =  .2013,  WEAPON     1 VS. AIRCRAFT TYPE    1.  ANNULAR RADII ARE    9.9139 AND     2.2369
ANNULUS PK =  .1594, CIRCULAR PK =  .1578,  WEAPON     2 VS. AIRCRAFT TYPE    1.  ANNULAR RADII ARE   12.1340 AND     3.4901

DISTANCE TO TARGET    3 FROM SUB LOCATION    2 IS    894.71 NM. FLIGHT TIME (MISSILE TYPE  1) =  6.73436818E+00 MIN.
WEAPON     3 ARRIVES ON TARGET     3 WHEN FIRST AIRCRAFT IS     10.31 NAUTICAL MILES BEYOND CENTROID.
ANNULUS PK =  .1886, CIRCULAR PK =  .1922,  WEAPON     3 VS. AIRCRAFT TYPE    1.  ANNULAR RADII ARE   10.3146 AND     2.3903

DISTANCE TO TARGET    3 FROM SUB LOCATION    3 IS   1488.83 NM. FLIGHT TIME (MISSILE TYPE  1) =  9.70338104E+00 MIN.
WEAPON     5 ARRIVES ON TARGET     3 WHEN FIRST AIRCRAFT IS     28.16 NAUTICAL MILES BEYOND CENTROID.
```

```
TARGET     4 BRAKE RELEASE TIMES
AIRCRAFT TYPE =     1 STARTS AT    4.50 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    4.67 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    4.97 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.00 MINUTES.
AIRCRAFT TYPE =     1 STARTS AT    5.17 MINUTES.

DISTANCE TO TARGET   4 FROM SUB LOCATION   1 IS    579.79 NM. FLIGHT TIME (MISSILE TYPE  1) =  5.56297467E+00 MIN.
WEAPON    1 ARRIVES ON TARGET    4 WHEN FIRST AIRCRAFT IS       0.00 NAUTICAL MILES BEYOND CENTROID.

DISTANCE TO TARGET   4 FROM SUB LOCATION   2 IS    599.22 NM. FLIGHT TIME (MISSILE TYPE  1) =  5.64028240E+0, MIN.
WEAPON    3 ARRIVES ON TARGET    4 WHEN FIRST AIRCRAFT IS       0.00 NAUTICAL MILES BEYOND CENTROID.

DISTANCE TO TARGET   4 FROM SUB LOCATION   3 IS   1901.98 NM. FLIGHT TIME (MISSILE TYPE  1) =  1.02896268E+01 MIN.
WEAPON    5 ARRIVES ON TARGET    4 WHEN FIRST AIRCRAFT IS      37.24 NAUTICAL MILES BEYOND CENTROID.
ANNULUS PK =  .0517, CIRCULAR PK =  .0237,  WEAPON    5 VS. AIRCRAFT TYPE    1.  ANNULAR RADII ARE   37.2406 AND   31.1114
```

```
SURVIVABILITY MATRIX, AIRCRAFT TYPE    1

TARGET    1
  .9617  .9670
  .7791  .8284
  .9234  .9386

TARGET    2
  .9279  .9405
  .9254  .9387
  .9131  .9301

TARGET    3
  .8403  .8719
  .8468  .8783
  .9684  .9724

TARGET    4
  .1878  .1878
  .1878  .1878
  .9807  .9828

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST   .1000000000 TO CAUSE ITERATION.
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	1					
2	5	1	15.0000	3.3141	15.0000	3.3141
3	3	1	15.0000	4.5631	15.0000	4.5631
4	1	1	15.0000	3.0380	15.0000	3.0380
TOTALS		1	5.0000	4.0612	5.0000	4.0612
		1			50.0000	14.9765

EXPECTED VALUE KILLED = 14.9765

ITERATION NUMBER 1

DELTA N IN COLUMN 1 FROM ROW 2 TO ROW 3 IS 1.000

EXPECTED VALUE KILLED = 16.0766

ITERATION NUMBER 2

DELTA N IN COLUMN 2 FROM ROW 2 TO ROW 4 IS .475

EXPECTED VALUE KILLED = 16.2589

ITERATION NUMBER 3

DELTA N IN COLUMN 3 FROM ROW 3 TO ROW 1 IS 1.000

EXPECTED VALUE KILLED = 17.0229

ITERATION NUMBER 4

DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .474

EXPECTED VALUE KILLED = 17.2970

```
ITERATION NUMBER          5
DELTA N IN COLUMN    5 FROM ROW    3 TO ROW    2 IS      1.000
EXPECTED VALUE KILLED =    17.9057

ITERATION NUMBER          6
DELTA N IN COLUMN    6 FROM ROW    3 TO ROW    2 IS      1.000
EXPECTED VALUE KILLED =    14.2968

ITERATION NUMBER          7
DELTA N IN COLUMN    2 FROM ROW    2 TO ROW    3 IS       .525
EXPECTED VALUE KILLED =    18.8510

ITERATION NUMBER          8
DELTA N IN COLUMN    4 FROM ROW    2 TO ROW    1 IS      1.000
EXPECTED VALUE KILLED =    19.6168

ITERATION NUMBER          9
DELTA N IN COLUMN    3 FROM ROW    1 TO ROW    3 IS       .298
EXPECTED VALUE KILLED =    19.6509

ITERATION NUMBER         10
DELTA N IN COLUMN    4 FROM ROW    1 TO ROW    4 IS       .061
EXPECTED VALUE KILLED =    19.6559

ITERATION NUMBER         11
DELTA N IN COLUMN    2 FROM ROW    4 TO ROW    5 IS       .043
EXPECTED VALUE KILLED =    19.6584
```

```
ITERATION NUMBER        12
DELTA N IN COLUMN   4 FROM ROW   1 TO ROW   4 IS      .039
EXPECTED VALUE KILLED =   19.6605
LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST   .0100000000 TO CAUSE ITERATION.
```

```
ALLOCATION.     ITERATION NUMBER        13


TARGET     1
     1.7016 MISSILES FROM    2 SUBS AT SUB LOCATION    2. SALVO    1   (WEAPON     3).
     1.3740 MISSILES FROM    2 SUBS AT SUB LOCATION    2. SALVO    2   (WEAPON     4).

TARGET     2
     2.0000 MISSILES FROM    2 SUBS AT SUB LOCATION    3. SALVO    1   (WEAPON     5).
     2.0000 MISSILES FROM    2 SUBS AT SUB LOCATION    3. SALVO    2   (WEAPON     6).

TARGET     3
     1.0000 MISSILES FROM    1 SUBS AT SUB LOCATION    1. SALVO    1   (WEAPON     1).
      .5684 MISSILES FROM    1 SUBS AT SUB LOCATION    1. SALVO    2   (WEAPON     2).
      .2984 MISSILES FROM    2 SUBS AT SUB LOCATION    2. SALVO    1   (WEAPON     3).

TARGET     4
      .4316 MISSILES FROM    1 SUBS AT SUB LOCATION    1. SALVO    2   (WEAPON     2).
      .6260 MISSILES FROM    2 SUBS AT SUB LOCATION    2. SALVO    2   (WEAPON     4).

SUB LOCATION     1
     1.0000 MISSILES FROM SALVO    1 TO TARGET    3
      .5684 MISSILES FROM SALVO    2 TO TARGET    3
      .4316 MISSILES FROM SALVO    2 TO TARGET    4

SUB LOCATION     2
     1.7016 MISSILES FROM SALVO    1 TO TARGET    1
      .2984 MISSILES FROM SALVO    1 TO TARGET    3
     1.3740 MISSILES FROM SALVO    2 TO TARGET    1
      .6260 MISSILES FROM SALVO    2 TO TARGET    4

SUB LOCATION     3
     2.0000 MISSILES FROM SALVO    1 TO TARGET    2
     2.0000 MISSILES FROM SALVO    2 TO TARGET    2
```

MULTIPLIER MATRIX

```
TARGET     1
  .2956  .2544
 1.3905 1.4260
  .6'37  .4802

TARGET     2
  .8093  .6633
  .8390  .6847
  [illegible]  .7838

TARGET     3
 1.9308 1.5213
 1.8444 1.4400
  .3562  .3104

TARGET     4
 1.4260 1.4260
 1.4260 1.4260
  .0166  .0148
```

ITERATION NUMBER 14

DELTA N IN COLUMN 2 FROM ROW 4 TO ROW 3 IS .036

EXPECTED VALUE KILLED = 19.6622

ITERATION NUMBER 15

DELTA N IN COLUMN 3 FROM ROW 3 TO ROW 1 IS .071

EXPECTED VALUE KILLED = 19.6641

ITERATION NUMBER 16

DELTA N IN COLUMN 4 FROM ROW 1 TO ROW 4 IS .042

EXPECTED VALUE KILLED = 19.6665

```
ITERATION NUMBER          17
DELTA N IN COLUMN   2 FROM ROW   4 TO ROW   3 IS      .045
EXPECTED VALUE KILLED =   19.6692

ITERATION NUMBER          18
DELTA N IN COLUMN   3 FROM ROW   3 TO ROW   1 IS      .034
EXPECTED VALUE KILLED =   19.6696

ITERATION NUMBER          19
DELTA N IN COLUMN   4 FROM ROW   1 TO ROW   4 IS      .045
EXPECTED VALUE KILLED =   19.6724

ITERATION NUMBER          20
DELTA N IN COLUMN   2 FROM ROW   4 TO ROW   3 IS      .045
EXPECTED VALUE KILLED =   19.6751

ITERATION NUMBER          21
DELTA N IN COLUMN   3 FROM ROW   3 TO ROW   1 IS      .035
EXPECTED VALUE KILLED =   19.6755

ITERATION NUMBER          22
DELTA N IN COLUMN   4 FROM ROW   1 TO ROW   4 IS      .045
EXPECTED VALUE KILLED =   19.6783

ITERATION NUMBER          23
DELTA N IN COLUMN   2 FROM ROW   4 TO ROW   3 IS      .045
EXPECTED VALUE KILLED =   19.6809
```

```
ITERATION NUMBER        24
DELTA N IN COLUMN   3 FROM ROW   3 TO ROW   1 IS      .035
EXPECTED VALUE KILLED =   19.6814

ITERATION NUMBER        25
DELTA N IN COLUMN   4 FROM ROW   1 TO ROW   4 IS      .045
EXPECTED VALUE KILLED =   19.6841

ITERATION NUMBER        26
DELTA N IN COLUMN   2 FROM ROW   4 TO ROW   3 IS      .045
EXPECTED VALUE KILLED =   19.6868

ITERATION NUMBER        27
DELTA N IN COLUMN   3 FROM ROW   3 TO ROW   1 IS      .035
EXPECTED VALUE KILLED =   19.6873

ITERATION NUMBER        28
DELTA N IN COLUMN   4 FROM ROW   1 TO ROW   4 IS      .045
EXPECTED VALUE KILLED =   19.6900

ITERATION NUMBER        29
DELTA N IN COLUMN   2 FROM ROW   4 TO ROW   3 IS      .045
EXPECTED VALUE KILLED =   19.6927

ITERATION NUMBER        30
DELTA N IN COLUMN   3 FROM ROW   3 TO ROW   1 IS      .035
EXPECTED VALUE KILLED =   19.6932
```

```
ITERATION NUMBER        31
DELTA N IN COLUMN   4 FROM ROW   1 TO ROW   4 IS      .045
EXPECTED VALUE KILLED =   19.6959

ITERATION NUMBER        32
DELTA N IN COLUMN   2 FROM ROW   4 TO ROW   3 IS      .045
EXPECTED VALUE KILLED =   19.6986

ITERATION NUMBER        33
DELTA N IN COLUMN   3 FROM ROW   3 TO ROW   1 IS      .035
EXPECTED VALUE KILLED =   19.6990

ITERATION NUMBER        34
DELTA N IN COLUMN   4 FROM ROW   1 TO ROW   4 IS      .045
EXPECTED VALUE KILLED =   19.7017

ITERATION NUMBER        35
DELTA N IN COLUMN   2 FROM ROW   4 TO ROW   3 IS      .045
EXPECTED VALUE KILLED =   19.7044

ITERATION NUMBER        36
DELTA N IN COLUMN   3 FROM ROW   3 TO ROW   1 IS      .018
EXPECTED VALUE KILLED =   19.7048

ITERATION NUMBER        37
DELTA N IN COLUMN   4 FROM ROW   1 TO ROW   4 IS      .043
EXPECTED VALUE KILLED =   19.7072
```

```
ITERATION NUMBER        38
DELTA N IN COLUMN   2 FROM ROW   4 TO ROW   3 IS      .041
EXPECTED VALUE KILLED =   19.7095

ITERATION NUMBER        39
DELTA N IN COLUMN   4 FROM ROW   1 TO ROW   4 IS      .037
EXPECTED VALUE KILLED =   19.7113

ITERATION NUMBER        40
DELTA N IN COLUMN   2 FROM ROW   4 TO ROW   3 IS      .034
EXPECTED VALUE KILLED =   19.7129

ITERATION NUMBER        41
DELTA N IN COLUMN   4 FROM ROW   1 TO ROW   4 IS      .031
EXPECTED VALUE KILLED =   19.7142

ITERATION NUMBER        42
DELTA N IN COLUMN   2 FROM ROW   4 TO ROW   3 IS      .007
EXPECTED VALUE KILLED =   19.7147

ITERATION NUMBER        43
DELTA N IN COLUMN   4 FROM ROW   1 TO ROW   4 IS      .006
EXPECTED VALUE KILLED =   19.7147
LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST   .0010000000 TO CAUSE ITERATION.
```

```
ALLOCATION.     ITERATION NUMBER        44


TARGET     1
      2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
       .9460 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

TARGET     2
      2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   3. SALVO   1  (WEAPON    5).
      2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   3. SALVO   2  (WEAPON    6).

TARGET     3
      1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
      1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).

TARGET     4
      1.0540 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

SUB LOCATION     1
      1.0000 MISSILES FROM SALVO   1 TO TARGET   3
      1.0000 MISSILES FROM SALVO   2 TO TARGET   3

SUB LOCATION     2
      2.0000 MISSILES FROM SALVO   1 TO TARGET   1
       .9460 MISSILES FROM SALVO   2 TO TARGET   1
      1.0540 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION     3
      2.0000 MISSILES FROM SALVO   1 TO TARGET   2
      2.0000 MISSILES FROM SALVO   2 TO TARGET   2
```

```
MULTIPLIER MATRIX


TARGET     1
  .2974  .2560
 1.9021 1.4347
  .6074  .4831

TARGET     2
  .8093  .6633
  .8390  .6847
  .9833  .7838

TARGET     3
 1.9124 1.5068
 1.8268 1.4263
  .3528  .3074

TARGET     4
 1.4347 1.4347
 1.4347 1.4347
  .0167  .0149

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST   .0001000000 TO CAUSE ITERATION.
```

```
ALLOCATION.     ITERATION NUMBER        45


TARGET    1
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
      .9460 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

TARGET    2
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   3. SALVO   1  (WEAPON    5).
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   3. SALVO   2  (WEAPON    6).

TARGET    3
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).

TARGET    4
     1.0540 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

SUB LOCATION    1
     1.0000 MISSILES FROM SALVO   1 TO TARGET   3
     1.0000 MISSILES FROM SALVO   2 TO TARGET   3

SUB LOCATION    2
     2.0000 MISSILES FROM SALVO   1 TO TARGET   1
      .9460 MISSILES FROM SALVO   2 TO TARGET   1
     1.0540 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION    3
     2.0000 MISSILES FROM SALVO   1 TO TARGET   2
     2.0000 MISSILES FROM SALVO   2 TO TARGET   2
```

```
MULTIPLIER MATRIX


TARGET    1
  .2974  .2560
 1.9021 1.4347
  .6074  .4831

TARGET    2
  .8093  .6633
  .8390  .6847
  .9833  .7838

TARGET    3
 1.9124 1.5068
 1.8268 1.4263
  .3528  .3074

TARGET    4
 1.4347 1.4347
 1.4347 1.4347
  .0167  .0149

MULTIPLIER MATRIX CONVERGED WITHIN TOLERANCE OF  .0001000000

CURRENT DELTA LAMBDA IS  .0001000000
```

```
ALLOCATION.     ITERATION NUMBER        46

TARGET     1
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
      .9460 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

TARGET     2
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   3. SALVO   1  (WEAPON    5).
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   3. SALVO   2  (WEAPON    6).

TARGET     3
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).

TARGET     4
     1.0540 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAP      4).

SUB LOCATION     1
     1.0000 MISSILES FROM SALVO   1 TO TARGET   3
     1.0000 MISSILES FROM SALVO   2 TO TARGET   3

SUB LOCATION     2
     2.0000 MISSILES FROM SALVO   1 TO TARGET   1
      .9460 MISSILES FROM SALVO   2 TO TARGET   1
     1.0540 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION     3
     2.0000 MISSILES FROM SALVO   1 TO TARGET   2
     2.0000 MISSILES FROM SALVO   2 TO TARGET   2
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	3	1	15.0000	7.3817	15.0000	7.3817
2	4	1	15.0000	4.1798	15.0000	4.1798
3	2	1	15.0000	4.0110	15.0000	4.0110
4	1	1	5.0000	4.1423	5.0000	4.1423
TOTALS		1			50.0000	19.7147

EXPECTED VALUE KILLED = 19.7147

MULTIPLIER MATRIX

```
TARGET     1
  .2974  .2560
 1.9021 1.4347
  .6074  .4831

TARGET     2
  .8093  .6633
  .8390  .6847
  .9833  .7838

TARGET     3
 1.9124 1.5068
 1.8268 1.4263
  .3528  .3074

TARGET     4
 1.4347 1.4347
 1.4347 1.4347
  .0157  .0149
```

```
FOR MISSILE TYPE    1
SUM OF LOWEST LAMBDAS WITH WEAPONS =     1.7671
SUM OF HIGHEST LAMBDAS =     3.4193
SUB MOVED FROM LOCATION     3 TO LOCATION     1
THIS IS MOVE NUMBER     1 OF A SUB.
```

```
ALLOCATION.     ITERATION NUMBER         0


TARGET     1
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
      .9460 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

TARGET     2
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   1  (WEAPON    5).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   2  (WEAPON    6).

TARGET     3
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).

TARGET     4
     1.0540 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

SUB LOCATION     1
     2.0000 MISSILES FROM SALVO   1 TO TARGET   3
     2.0000 MISSILES FROM SALVO   2 TO TARGET   3

SUB LOCATION     2
     2.0000 MISSILES FROM SALVO   1 TO TARGET   1
      .9460 MISSILES FROM SALVO   2 TO TARGET   1
     1.0540 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION     3
     1.0000 MISSILES FROM SALVO   1 TO TARGET   2
     1.0000 MISSILES FROM SALVO   2 TO TARGET   2

SUB POINT NUMBER     NUMBER OF SUBS     SUB TYPE
       1                   2               1
       2                   2               1
       3                   1               1

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST   .1000000000 TO CAUSE ITERATION.
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	3					
		1	15.0000	7.3817	15.0000	7.3817
2	2					
		1	15.0000	2.2602	15.0000	2.2602
3	4					
		1	15.0000	6.9494	15.0000	6.9494
4	1					
		1	5.0000	4.1423	5.0000	4.1423
TOTALS						
		1			50.0000	20.7335

EXPECTED VALUE KILLED = 20.7335

ITERATION NUMBER 47

DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .145

EXPECTED VALUE KILLED = 20.7566

ITERATION NUMBER 48

DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .130

EXPECTED VALUE KILLED = 20.7773

ITERATION NUMBER 49

DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .120

EXPECTED VALUE KILLED = 20.7933

ITERATION NUMBER 50

DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .108

EXPECTED VALUE KILLED = 20.8074

```
ITERATION NUMBER        51
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .100
EXPECTED VALUE KILLED =   20.8185

ITERATION NUMBER        52
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .090
EXPECTED VALUE KILLED =   20.8282

ITERATION NUMBER        53
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .083
EXPECTED VALUE KILLED =   20.8358

ITERATION NUMBER        54
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .075
EXPECTED VALUE KILLED =   20.8425

ITERATION NUMBER        55
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .069
EXPECTED VALUE KILLED =   20.8477

ITERATION NUMBER        56
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .062
EXPECTED VALUE KILLED =   20.8523

ITERATION NUMBER        57
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .057
EXPECTED VALUE KILLED =   20.8560
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```
ITERATION NUMBER          58
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS       .051
EXPECTED VALUE KILLED =   20.8591

ITERATION NUMBER          59
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS       .048
EXPECTED VALUE KILLED =   20.8616
LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST   .0100000000 TO CAUSE ITERATION.
```

```
ALLOCATION.     ITERATION NUMBER        60


TARGET     1
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
     1.4622 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

TARGET     2
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   1  (WEAPON    5).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   2  (WEAPON    6).

TARGET     3
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
     1.3781 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).

TARGET     4
      .6219 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).
      .5378 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

SUB LOCATION     1
     2.0000 MISSILES FROM SALVO    1 TO TARGET    3
     1.3781 MISSILES FROM SALVO    2 TO TARGET    3
      .6219 MISSILES FROM SALVO    2 TO TARGET    4

SUB LOCATION     2
     2.0000 MISSILES FROM SALVO    1 TO TARGET    1
     1.4622 MISSILES FROM SALVO    2 TO TARGET    1
      .5378 MISSILES FROM SALVO    2 TO TARGET    4

SUB LOCATION     3
     1.0000 MISSILES FROM SALVO    1 TO TARGET    2
     1.0000 MISSILES FROM SALVO    2 TO TARGET    2
```

MULTIPLIER MATRIX

```
TARGET    1
  .2699  .2323
 1.7259 1.3018
  .5511  .4384

TARGET    2
  .9529  .7810
  .9878  .8062
 1.1578  .9228

TARGET    3
 1.5258 1.2022
 1.4575 1.1379
  .2815  .2453

TARGET    4
 1.2022 1.2022
 1.2022 1.2022
  .0140  .0125

ITERATION NUMBER        61

DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .043

EXPECTED VALUE KILLED =   20.8638

ITERATION NUMBER        62

DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .040

EXPECTED VALUE KILLED =   20.8655

ITERATION NUMBER        63

DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .036

EXPECTED VALUE KILLED =   20.8670
```

```
ITERATION NUMBER         64
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .033
EXPECTED VALUE KILLED =   20.8682

ITERATION NUMBER         65
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .030
EXPECTED VALUE KILLED =   20.8692

ITERATION NUMBER         66
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .027
EXPECTED VALUE KILLED =   20.8701

ITERATION NUMBER         67
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .025
EXPECTED VALUE KILLED =   20.8708

ITERATION NUMBER         68
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .023
EXPECTED VALUE KILLED =   20.8713

ITERATION NUMBER         69
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .020
EXPECTED VALUE KILLED =   20.8718

ITERATION NUMBER         70
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .019
EXPECTED VALUE KILLED =   20.8722
```

```
ITERATION NUMBER          71
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .017
EXPECTED VALUE KILLED =   20.8725

ITERATION NUMBER          72
DELTA N IN COLUMN   [illegible] FROM ROW   3 TO ROW   4 IS      .016
EXPECTED VALUE KILL        20.8728

ITERATION NUMBER          73
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .014
EXPECTED VALUE KILLED =   20.8730

ITERATION NUMBER          74
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .013
EXPECTED VALUE KILLED =   20.8732

ITERATION NUMBER          75
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .012
EXPECTED VALUE KILLED =   20.8734

ITERATION NUMBER          76
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .011
EXPECTED VALUE KILLED =   20.8735

ITERATION NUMBER          77
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .010
EXPECTED VALUE KILLED =   20.8736
```

```
ITERATION NUMBER        78
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .009
EXPECTED VALUE KILLED =   20.8737

ITERATION NUMBER        79
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .008
EXPECTED VALUE KILLED =   20.8738

ITERATION NUMBER        80
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .007
EXPECTED VALUE KILLED =   20.8739

ITERATION NUMBER        81
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .007
EXPECTED VALUE KILLED =   20.8739

ITERATION NUMBER        82
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .006
EXPECTED VALUE KILLED =   20.8740

ITERATION NUMBER        83
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .006
EXPECTED VALUE KILLED =   20.8740

ITERATION NUMBER        84
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .005
EXPECTED VALUE KILLED =   20.8740
```

```
ITERATION NUMBER          85

DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .005

EXPECTED VALUE KILLED =   20.8740

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST   .0010000000 TO CAUSE ITERATION.
```

```
ALLOCATION.     ITERATION NUMBER        86


TARGET    1
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
     1.6922 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

TARGET    2
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   1  (WEAPON    5).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   2  (WEAPON    6).

TARGET    3
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
     1.1698 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).

TARGET    4
      .8302 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).
      .3078 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

SUB LOCATION    1
     2.0000 MISSILES FROM SALVO   1 TO TARGET    3
     1.1698 MISSILES FROM SALVO   2 TO TARGET    3
      .8302 MISSILES FROM SALVO   2 TO TARGET    4

SUB LOCATION    2
     2.0000 MISSILES FROM SALVO   1 TO TARGET    1
     1.6922 MISSILES FROM SALVO   2 TO TARGET    1
      .3078 MISSILES FROM SALVO   2 TO TARGET    4

SUB LOCATION    3
     1.0000 MISSILES FROM SALVO   1 TO TARGET    2
     1.0000 MISSILES FROM SALVO   2 TO TARGET    2
```

```
MULTIPLIER MATRIX


TARGET     1
  .2585  .2224
 1.6527 1.2466
  .5278  .4198

TARGET     2
  .9529  .7810
  .9878  .8062
 1.1578  .9228

TARGET     3
 1.5700 1.2370
 1.4997 1.1709
  .2896  .2524

TARGET     4
 1.2466 1.2466
 1.2466 1.2466
  .0145  .0130

ITERATION NUMBER         87

DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .004

EXPECTED VALUE KILLED =    20.8741

ITERATION NUMBER         88

DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .004

EXPECTED VALUE KILLED =    20.8741

ITERATION NUMBER         89

DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .004

EXPECTED VALUE KILLED =    20.8741
```

```
ITERATION NUMBER        90
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .003
EXPECTED VALUE KILLED =   20.8741

ITERATION NUMBER        91
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .003
EXPECTED VALUE KILLED =   20.8741

ITERATION NUMBER        92
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .003
EXPECTED VALUE KILLED =   20.8741

ITERATION NUMBER        93
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .002
EXPECTED VALUE KILLED =   20.8741

ITERATION NUMBER        94
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .002
EXPECTED VALUE KILLED =   20.8741

ITERATION NUMBER        95
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .002
EXPECTED VALUE KILLED =   20.8741

ITERATION NUMBER        96
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .002
EXPECTED VALUE KILLED =   20.8741
```

```
ITERATION NUMBER        97
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .002
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        98
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .002
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        99
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .001
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER       100
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .001
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER       101
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .001
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER       102
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .004
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER       103
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .001
EXPECTED VALUE KILLED =   20.8742
```

```
ITERATION NUMBER        104
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS       .001
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        105
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS       .001
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        106
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS       .001
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        107
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS       .001
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        108
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS       .001
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        109
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS       .001
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        110
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS       .000
EXPECTED VALUE KILLED =   20.8742
```

```
ITERATION NUMBER       111

DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .000

EXPECTED VALUE KILLED =   20.8742

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST   .0001000000 TO CAUSE ITERATION.
```

```
ALLOCATION,     ITERATION NUMBER      112

TARGET    1
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2, SALVO   1  (WEAPON    3).
     1.7124 MISSILES FROM   2 SUBS AT SUB LOCATION   2, SALVO   2  (WEAPON    4).

TARGET    2
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3, SALVO   1  (WEAPON    5).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3, SALVO   2  (WEAPON    6).

TARGET    3
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1, SALVO   1  (WEAPON    1).
     1.1468 MISSILES FROM   2 SUBS AT SUB LOCATION   1, SALVO   2  (WEAPON    2).

TARGET    4
      .8532 MISSILES FROM   2 SUBS AT SUB LOCATION   1, SALVO   2  (WEAPON    2).
      .2876 MISSILES FROM   2 SUBS AT SUB LOCATION   2, SALVO   2  (WEAPON    4).

SUB LOCATION    1
     2.0000 MISSILES FROM SALVO   1 TO TARGET   3
     1.1468 MISSILES FROM SALVO   2 TO TARGET   3
      .8532 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION    2
     2.0000 MISSILES FROM SALVO   1 TO TARGET   1
     1.7124 MISSILES FROM SALVO   2 TO TARGET   1
      .2876 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION    3
     1.0000 MISSILES FROM SALVO   1 TO TARGET   2
     1.0000 MISSILES FROM SALVO   2 TO TARGET   2
```

```
MULTIPLIER MATRIX

TARGET     1
  .2575  .2216
 1.6464 1.2419
  .5258  .4182

TARGET     2
  .9529  .7810
  .9878  .8062
 1.1578  .9228

TARGET     3
 1.5749 1.2409
 1.5044 1.1746
  .2905  .2532

TARGET     4
 1.2409 1.2409
 1.2409 1.2409
  .0144  .0129

ITERATION NUMBER       113

DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .000

EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER       114

DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .000

EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER       115

DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .000

EXPECTED VALUE KILLED =   20.8742
```

ITERATION NUMBER 116

DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .000

EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 117

DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .000

EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 118

DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .000

EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 119

DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .000

EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 120

DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .000

EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 121

DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .000

EXPECTED VALUE KILLED = 20.8742

ITERATION NUMBER 122

DELTA N IN COLUMN 2 FROM ROW 3 TO ROW 4 IS .000

EXPECTED VALUE KILLED = 20.8742

```
ITERATION NUMBER        123
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .000
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        124
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .000
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        125
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .000
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        126
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .000
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        127
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .000
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        128
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .000
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        129
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .000
EXPECTED VALUE KILLED =   20.8742
```

```
ITERATION NUMBER        130
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .000
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        131
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .000
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        132
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .000
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        133
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .000
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        134
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .000
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        135
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .000
EXPECTED VALUE KILLED =   20.8742

ITERATION NUMBER        136
DELTA N IN COLUMN   2 FROM ROW   3 TO ROW   4 IS      .000
EXPECTED VALUE KILLED =   20.8742
```

```
ITERATION NUMBER        137
DELTA N IN COLUMN   4 FROM ROW   4 TO ROW   1 IS      .000
EXPECTED VALUE KILLED =   20.8742
MULTIPLIER MATRIX CONVERGED WITHIN TOLERANCE OF  .0001000000
CURRENT DELTA LAMBDA IS  .0001000000
```

```
ALLOCATION.     ITERATION NUMBER      138

TARGET    1
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
     1.7147 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

TARGET    2
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   1  (WEAPON    5).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   2  (WEAPON    6).

TARGET    3
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
     1.1448 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).

TARGET    4
      .8552 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).
      .2853 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

SUB LOCATION    1
     2.0000 MISSILES FROM SALVO   1 TO TARGET   3
     1.1448 MISSILES FROM SALVO   2 TO TARGET   3
      .8552 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION    2
     2.0000 MISSILES FROM SALVO   1 TO TARGET   1
     1.7147 MISSILES FROM SALVO   2 TO TARGET   1
      .2853 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION    3
     1.0000 MISSILES FROM SALVO   1 TO TARGET   2
     1.0000 MISSILES FROM SALVO   2 TO TARGET   2
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	4	1	15.0000	8.4083	15.0000	8.4083
2	2	1	15.0000	2.2602	15.0000	2.2602
3	3	1	15.0000	5.9478	15.0000	5.9478
4	1	1	5.0000	4.2579	5.0000	4.2579
TOTALS		1			50.0000	20.8742

EXPECTED VALUE KILLED = 20.8742

MULTIPLIER MATRIX

TARGET	1
.2574	.2215
1.6457	1.2413
.5255	.4180
TARGET	2
.9529	.7810
.9878	.8062
1.1578	.9228
TARGET	3
1.5754	1.2412
1.5049	1.1749
.2906	.2532
TARGET	4
1.2413	1.2413
1.2413	1.2413
.0144	.0129

```
FOR MISSILE TYPE    1
SUM OF LOWEST LAMBDAS WITH WEAPONS =    2.0806
SUM OF HIGHEST LAMBDAS =    2.9871
SUB MOVED FROM LOCATION    3 TO LOCATION    2
THIS IS MOVE NUMBER    2 OF A SUB.
```

```
ALLOCATION.     ITERATION NUMBER         0


TARGET    1
     3.0000 MISSILES FROM   3 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
     1.7147 MISSILES FROM   3 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

TARGET    2

TARGET    3
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
     1.1448 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).

TARGET    4
      .8552 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).
     1.2853 MISSILES FROM   3 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

SUB LOCATION    1
     2.0000 MISSILES FROM SALVO   1 TO TARGET   3
     1.1448 MISSILES FROM SALVO   2 TO TARGET   3
      .8552 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION    2
     3.0000 MISSILES FROM SALVO   1 TO TARGET   1
     1.7147 MISSILES FROM SALVO   2 TO TARGET   1
     1.2853 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION    3

SUB POINT NUMBER     NUMBER OF SUBS     SUB TYPE
       1                   2                1
       2                   3                1
       3                   0                1

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST   .1000000000 TO CAUSE ITERATION.
```

```
ALLOCATION.     ITERATION NUMBER       155


TARGET    1
     3.0000 MISSILES FROM   3 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
     1.4804 MISSILES FROM   3 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

TARGET    2

TARGET    3
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).
      .2562 MISSILES FROM   3 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

TARGET    4
     1.2634 MISSILES FROM   3 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

SUB LOCATION    1
     2.0000 MISSILES FROM SALVO   1 TO TARGET   3
     2.0000 MISSILES FROM SALVO   2 TO TARGET   3

SUB LOCATION    2
     3.0000 MISSILES FROM SALVO   1 TO TARGET   1
     1.4804 MISSILES FROM SALVO   2 TO TARGET   1
      .2562 MISSILES FROM SALVO   2 TO TARGET   3
     1.2634 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION    3
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	4	1	15.0000	9.6330	15.0000	9.6330
2	0	1	15.0000	0.0000	15.0000	0.0000
3	4	1	15.0000	7.2128	15.0000	7.2128
4	i	1	5.0000	4.3957	5.0000	4.3957
TOTALS		1			50.0000	21.2415

EXPECTED VALUE KILLED = 21.2415

MULTIPLIER MATRIX

```
TARGET     1
  .2095  .1803
 1.3400 1.0107
  .4279  .3404

TARGET     2
 1.1219  .9196
 1.1631  .9493
 1.3632 1.0866

TARGET     3
 1.3552 1.0678
 1.2946 1.0107
  .2500  .2178

TARGET     4
 1.0107 1.0107
 1.0107 1.0107
  .0118  .0105
```

```
FOR MISSILE TYPE    1
SUM OF LOWEST LAMBDAS WITH WEAPONS =    2.3507
SUM OF HIGHEST LAMBDAS =    2.4498
SUB MOVED FROM LOCATION    2 TO LOCATION    3
THIS IS MOVE NUMBER    3 OF A SUB.
```

```
ALLOCATION.     ITERATION NUMBER        0


TARGET    1
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
      .4804 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

TARGET    2
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   1  (WEAPON    5).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   2  (WEAPON    6).

TARGET    3
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).
      .2562 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

TARGET    4
     1.2634 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

SUB LOCATION    1
     2.0000 MISSILES FROM SALVO   1 TO TARGET   3
     2.0000 MISSILES FROM SALVO   2 TO TARGET   3

SUB LOCATION    2
     2.0000 MISSILES FROM SALVO   1 TO TARGET   1
      .4804 MISSILES FROM SALVO   2 TO TARGET   1
      .2562 MISSILES FROM SALVO   2 TO TARGET   3
     1.2634 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION    3
     1.0000 MISSILES FROM SALVO   1 TO TARGET   2
     1.0000 MISSILES FROM SALVO   2 TO TARGET   2

SUB POINT NUMBER     NUMBER OF SUBS     SUB TYPE
       1                   2                1
       2                   2                1
       3                   1                1

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST   .1000000000 TO CAUSE ITERATION.
```

```
ALLOCATION.     ITERATION NUMBER       249

TARGET    1
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
     1.7147 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

TARGET    2
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   1  (WEAPON    5).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   2  (WEAPON    6).

TARGET    3
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
     1.1448 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).

TARGET    4
      .8552 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).
      .2853 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

SUB LOCATION    1
     2.0000 MISSILES FROM SALVO   1 TO TARGET   3
     1.1448 MISSILES FROM SALVO   2 TO TARGET   3
      .8552 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION    2
     2.0000 MISSILES FROM SALVO   1 TO TARGET   1
     1.7147 MISSILES FROM SALVO   2 TO TARGET   1
      .2853 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION    3
     1.0000 MISSILES FROM SALVO   1 TO TARGET   2
     1.0000 MISSILES FROM SALVO   2 TO TARGET   2
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	4	1	15.0000	8.4083	15.0000	8.4083
2	2	1	15.0000	2.2602	15.0000	2.2602
3	3	1	15.0000	5.9478	15.0000	5.9478
4	1	1	5.0000	4.2579	5.0000	4.2579
TOTALS		1			50.0000	20.8742

EXPECTED VALUE KILLED = 20.8742

```
SURVIVABILITY PRODUCT FOR TARGET    1. AIRCRAFT TYPE    1 =     .4394
SURVIVABILITY PRODUCT FOR TARGET    2. AIRCRAFT TYPE    1 =     .8493
SURVIVABILITY PRODUCT FOR TARGET    3. AIRCRAFT TYPE    1 =     .6035
SURVIVABILITY PRODUCT FOR TARGET    4. AIRCRAFT TYPE    1 =     .1484
    3.5045 AIRCRAFT OF TYPE    1 MOVED FROM TARGET    4 TO TARGET    2
THIS IS VALUE SHIFT NUMBER    1

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST   .1000000000 TO CAUSE ITERATION.
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	4	1	15.0000	8.4083	15.0000	8.4083
2	2	1	18.5045	2.7882	18.5045	2.7882
3	3	1	15.0000	5.9478	15.0000	5.9478
4	1	1	1.4955	1.2736	1.4955	1.2736
TOTALS		1			50.0000	18.4179

EXPECTED VALUE KILLED = 18.4179

ITERATION NUMBER 250

DELTA N IN COLUMN 2 FROM ROW 4 TO ROW 3 IS .667

EXPECTED VALUE KILLED = 18.7537

ITERATION NUMBER 251

DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 1 IS .049

EXPECTED VALUE KILLED = 18.7564

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .0100000000 TO CAUSE ITERATION.

```
ALLOCATION.      ITERATION NUMBER        298

TARGET     1
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

TARGET     2
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   1  (WEAPON    5).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   2  (WEAPON    6).

TARGET     3
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
     1.5480 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).

TARGET     4
      .4520 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).

SUB LOCATION     1
     2.0000 MISSILES FROM SALVO   1 TO TARGET   3
     1.5480 MISSILES FROM SALVO   2 TO TARGET   3
      .4520 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION     2
     2.0000 MISSILES FROM SALVO   1 TO TARGET   1
     2.0000 MISSILES FROM SALVO   2 TO TARGET   1

SUB LOCATION     3
     1.0000 MISSILES FROM SALVO   1 TO TARGET   2
     1.0000 MISSILES FROM SALVO   2 TO TARGET   2
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	4					
2	2	1	15.0000	8.7532	15.0000	8.7532
3	4	1	18.5045	2.7882	18.5045	2.7882
4	0	1	15.0000	6.4347	15.0000	6.4347
TOTALS		1	1.4955	.7934	1.4955	.7934
		1			50.0000	18.7694

EXPECTED VALUE KILLED = 18.7694

```
SURVIVABILITY PRODUCT FOR TARGET     1. AIRCRAFT TYPE    1 =     .4165
SURVIVABILITY PRODUCT FOR TARGET     2. AIRCRAFT TYPE    1 =     .8493
SURVIVABILITY PRODUCT FOR TARGET     3. AIRCRAFT TYPE    1 =     .5710
SURVIVABILITY PRODUCT FOR TARGET     4. AIRCRAFT TYPE    1 =     .4695
     6.4930 AIRCRAFT OF TYPE     1 MOVED FROM TARGET     1 TO TARGET     2
THIS IS VALUE SHIFT NUMBER     2

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST   .1000000000 TO CAUSE ITERATION.
```

TARGET NUMBER	WEAPONS ALLOCATED
1	4
2	2
3	4
4	0

	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
	1	8.5070	4.9642	8.5070	4.9642
	1	24.9975	3.7666	24.9975	3.7666
	1	15.0000	6.4347	15.0000	6.4347
	1	1.4955	.7934	1.4955	.7934
TOTALS	1			50.0000	15.9588

EXPECTED VALUE KILLED = 15.9588

ITERATION NUMBER 299

DELTA N IN COLUMN 3 FROM ROW 1 TO ROW 2 IS 1.898

EXPECTED VALUE KILLED = 16.7167

ITERATION NUMBER 300

DELTA N IN COLUMN 4 FROM ROW 1 TO ROW 4 IS .049

EXPECTED VALUE KILLED = 16.7192

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .0100000000 TO CAUSE ITERATION.

```
ALLOCATION.     ITERATION NUMBER       401

TARGET     1
     1.6170 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).

TARGET     2
      .3830 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
     1.8850 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   1  (WEAPON    5).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   2  (WEAPON    5).

TARGET     3
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
     1.6543 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).

TARGET     4
      .3457 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).
      .1150 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).

SUB LOCATION     1
     2.0000 MISSILES FROM SALVO   1 TO TARGET   3
     1.6543 MISSILES FROM SALVO   2 TO TARGET   3
      .3457 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION     2
     1.6170 MISSILES FROM SALVO   1 TO TARGET   1
      .3830 MISSILES FROM SALVO   1 TO TARGET   2
     1.8850 MISSILES FROM SALVO   2 TO TARGET   2
      .1150 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION     3
     1.0000 MISSILES FROM SALVO   1 TO TARGET   2
     1.0000 MISSILES FROM SALVO   2 TO TARGET   2
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	2	1	8.5070	2.8257	8.5070	2.8257
2	4	1	24.9975	6.7053	24.9975	6.7053
3	4	1	15.0000	6.5586	15.0000	6.5586
4	0	1	1.4955	.8035	1.4955	.8035
TOTALS		1			50.0000	16.8932

EXPECTED VALUE KILLED = 16.8932

```
SURVIVABILITY PRODUCT FOR TARGET    1. AIRCRAFT TYPE    1 =     .6678
SURVIVABILITY PRODUCT FOR TARGET    2. AIRCRAFT TYPE    1 =     .7318
SURVIVABILITY PRODUCT FOR TARGET    3. AIRCRAFT TYPE    1 =     .5628
SURVIVABILITY PRODUCT FOR TARGET    4. AIRCRAFT TYPE    1 =     .4627
     .4024 AIRCRAFT OF TYPE     1 MOVED FROM TARGET     4 TO TARGET     2
THIS IS VALUE SHIFT NUMBER     3

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST   .1000000000 TO CAUSE ITERATION.
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	2					
2	4	1	8.5070	2.8257	8.5070	2.8257
3	4	1	25.3998	6.8133	25.3998	6.8133
4	0	1	15.0000	6.5586	15.0000	6.5586
TOTALS		1	1.0932	.5873	1.0932	.5873
		1			50.0000	16.7849

EXPECTED VALUE KILLED = 16.7849

ITERATION NUMBER 402

DELTA N IN COLUMN 4 FROM ROW 4 TO ROW 2 IS .115

EXPECTED VALUE KILLED = 16.8124

ITERATION NUMBER 403

DELTA N IN COLUMN 2 FROM ROW 4 TO ROW 3 IS .067

EXPECTED VALUE KILLED = 16.8169

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .0100000000 TO CAUSE ITERATION.

```
ALLOCATION,     ITERATION NUMBER      408

TARGET    1
     1.5906 MISSILES FROM   2 SUBS AT SUB LOCATION   2, SALVO   1  (WEAPON    3).

TARGET    2
      .4094 MISSILES FROM   2 SUBS AT SUB LOCATION   2, SALVO   1  (WEAPON    3).
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2, SALVO   2  (WEAPON    4).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3, SALVO   1  (WEAPON    5).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3, SALVO   2  (WEAPON    6).

TARGET    3
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1, SALVO   1  (WEAPON    1).
     1.7212 MISSILES FROM   2 SUBS AT SUB LOCATION   1, SALVO   2  (WEAPON    2).

TARGET    4
      .2788 MISSILES FROM   2 SUBS AT SUB LOCATION   1, SALVO   2  (WEAPON    2).

SUB LOCATION    1
     2.0000 MISSILES FROM SALVO   1 TO TARGET   3
     1.7212 MISSILES FROM SALVO   2 TO TARGET   3
      .2788 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION    2
     1.5906 MISSILES FROM SALVO   1 TO TARGET   1
      .4094 MISSILES FROM SALVO   1 TO TARGET   2
     2.0000 MISSILES FROM SALVO   2 TO TARGET   2

SUB LOCATION    3
     1.0000 MISSILES FROM SALVO   1 TO TARGET   2
     1.0000 MISSILES FROM SALVO   2 TO TARGET   2
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUF	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	2	1	8.5070	2.7881	8.5070	2.7881
2	4	1	25.3998	6.9858	25.3998	6.9858
3	4	1	15.0000	6.6356	15.0000	6.6356
4	0	1	1.0932	.4075	1.0932	.4075
TOTALS		1			50.0000	16.8171

EXPECTED VALUE KILLED =   16.8171

```
SURVIVABILITY PRODUCT FOR TARGET    1. AIRCRAFT TYPE    1 =     .6423
SURVIVABILITY PRODUCT FOR TARGET    2. AIRCRAFT TYPE    1 =     .7250
SURVIVABILITY PRODUCT FOR TARGET    3. AIRCRAFT TYPE    1 =     .5576
SURVIVABILITY PRODUCT FOR TARGET    4. AIRCRAFT TYPE    1 =     .6273
    2.5101 AIRCRAFT OF TYPE    1 MOVED FROM TARGET    3 TO TARGET    2
THIS IS VALUE SHIFT NUMBER    4

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST   .1000000000 TO CAUSE ITERATION.
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	2	1	8.5070	[illegible]	8.5070	2.7881
2	4	1	27.9099	7.6762	27.9099	7.6762
3	4	1	12.4899	5.5252	12.4899	5.5252
4	0	1	1.0932	.4075	1.0932	.4075
TOTALS		1			50.0000	16.3970

EXPECTED VALUE KILLED = 16.3970

ITERATION NUMBER 409

DELTA N IN COLUMN 1 FROM ROW 3 TO ROW 2 IS .892

EXPECTED VALUE KILLED = 16.5333

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .0100000000 TO CAUSE ITERATION.

```
ALLOCATION.     ITERATION NUMBER      431

TARGET    1
     1.5460 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).

TARGET    2
     1.2985 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).
      .4540 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   1  (WEAPON    5).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   2  (WEAPON    6).

TARGET    3
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
      .4198 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).

TARGET    4
      .2817 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).
SUB LOCATION    1
     2.0000 MISSILES FROM SALVO   1 TO TARGET   3
     1.2985 MISSILES FROM SALVO   2 TO TARGET   2
      .4198 MISSILES FROM SALVO   2 TO TARGET   3
      .2817 MISSILES FROM SALVO   2 TO TARGET   4

SUB LOCATION    2
     1.5460 MISSILES FROM SALVO   1 TO TARGET   1
      .4540 MISSILES FROM SALVO   1 TO TARGET   2
     2.0000 MISSILES FROM SALVO   2 TO TARGET   2

SUB LOCATION    3
     1.0000 MISSILES FROM SALVO   1 TO TARGET   2
     1.0000 MISSILES FROM SALVO   2 TO TARGET   2
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	2					
		1	8.5070	2.7242	8.5070	2.7242
2	5					
		1	27.9099	9.2888	27.9099	9.2888
3	2					
		1	12.4899	4.1647	12.4899	4.1647
4	0					
		1	1.0932	.4107	1.0932	.4107
TOTALS						
		1			50.0000	16.5884

EXPECTED VALUE KILLED = 16.5884

```
SURVIVABILITY PRODUCT FOR TARGET    1. AIRCRAFT TYPE    1 =      .6798
SURVIVABILITY PRODUCT FOR TARGET    2. AIRCRAFT TYPE    1 =      .6672
SURVIVABILITY PRODUCT FO  TARGET    3. AIRCRAFT TYPE    1 =      .6666
SURVIVABILITY PRODUCT F(  TARGET    4. AIRCRAFT TYPE    1 =      .6243
     .0607 AIRCRAFT OF TYPE    1 MOVED FROM TARGET    4 TO TARGET    1
THIS IS VALUE SHIFT NUMBER    5

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST   .1000000000 TO CAUSE ITERATION.
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	2					
		1	8.5677	2.7436	8.5677	2.7436
2	5					
		1	27.9099	9.2888	27.9099	9.2888
3	2					
		1	12.4899	4.1647	12.4899	4.1647
4	0					
		1	1.0325	.3879	1.0325	.3879
TOTALS						
		1			50.0000	16.5851

EXPECTED VALUE KILLED = 16.5851

LAGRANGE MULTIPLIERS MUST DIFFER BY AT LEAST .0100000000 TO CAUSE ITERATION.

```
ALLOCATION.     ITERATION NUMBER      439

TARGET    1
     1.5739 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
TARGET    2
     1.3327 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).
      .4261 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   1  (WEAPON    5).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   2  (WEAPON    6).
TARGET    3
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
      .4198 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).
TARGET    4
      .2475 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).
SUB LOCATION    1
     2.0000 MISSILES FROM SALVO   1 TO TARGET   3
     1.3327 MISSILES FROM SALVO   2 TO TARGET   2
      .4198 MISSILES FROM SALVO   2 TO TARGET   3
      .2475 MISSILES FROM SALVO   2 TO TARGET   4
SUB LOCATION    2
     1.5739 MISSILES FROM SALVO   1 TO TARGET   1
      .4261 MISSILES FROM SALVO   1 TO TARGET   2
     2.0000 MISSILES FROM SALVO   2 TO TARGET   2
SUB LOCATION    3
     1.0000 MISSILES FROM SALVO   1 TO TARGET   2
     1.0000 MISSILES FROM SALVO   2 TO TARGET   2
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	2	1	8.5677	2.7840	8.5677	2.7840
2	5	1	27.9099	9.2876	27.9099	9.2876
3	2	1	12.4899	4.1647	12.4899	4.1647
4	0	1	1.0325	.3500	1.0325	.3500
TOTALS		1			50.0000	16.5863

EXPECTED VALUE KILLED = 16.5863

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SURVIVABILITY PRODUCT FOR TARGET    1. AIRCRAFT TYPE    1 =     .6751
SURVIVABILITY PRODUCT FOR TARGET    2. AIRCRAFT TYPE    1 =     .6672
SURVIVABILITY PRODUCT FOR TARGET    3. AIRCRAFT TYPE    1 =     .6666
SURVIVABILITY PRODUCT FOR TARGET    4. AIRCRAFT TYPE    1 =     .6610
     .0145 AIRCRAFT OF TYPE    1 MOVED FROM TARGET    4 TO TARGET    1
THIS IS VALUE SHIFT NUMBER    6
ALLOCATION INTEGERIZED
BEDDOWN INTEGERIZED
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```
ALLOCATION.     ITERATION NUMBER      439


TARGET    1
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   1  (WEAPON    3).

TARGET    2
     1.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   2. SALVO   2  (WEAPON    4).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   1  (WEAPON    5).
     1.0000 MISSILES FROM   1 SUBS AT SUB LOCATION   3. SALVO   2  (WEAPON    6).

TARGET    3
     2.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   1  (WEAPON    1).
     1.0000 MISSILES FROM   2 SUBS AT SUB LOCATION   1. SALVO   2  (WEAPON    2).

TARGET    4

SUB LOCATION    1
     2.0000 MISSILES FROM SALVO   1 TO TARGET   3
     1.0000 MISSILES FROM SALVO   2 TO TARGET   2
     1.0000 MISSILES FROM SALVO   2 TO TARGET   3

SUB LOCATION    2
     2.0000 MISSILES FROM SALVO   1 TO TARGET   1
     2.0000 MISSILES FROM SALVO   2 TO TARGET   2

SUB LOCATION    3
     1.0000 MISSILES FROM SALVO   1 TO TARGET   2
     1.0000 MISSILES FROM SALVO   2 TO TARGET   2
```

TARGET NUMBER	WEAPONS ALLOCATED	AIRCRAFT TYPE	TOTAL VALUE	KILLED VALUE	TOTAL AIRCRAFT	KILLED AIRCRAFT
1	2	1	9.0000	3.5376	9.0000	3.5376
2	5	1	28.0000	8.2922	28.0000	8.2922
3	3	1	12.0000	4.6130	12.0000	4.6130
4	0	1	1.0000	0.0000	1.0000	0.0000
TOTALS		1			50.0000	16.4428

EXPECTED VALUE KILLED = 16.4428

EOJ

```
06/15/73  SCOPE 3.2.0 - SCI  VER 113 43    10JUN73ECPA= 0754
11.26.05.WFPE34.
11.26.05.WFPE3.P4.T240.C060000.
11.26.05.TASK(PEAY.88090404-90C.SAR.2295)
11.26.06.RFL(44000)
11.26.06.COMMON(SAMCEM)
11.26.06.UPDATE(Q.P=SAMCEM.D.L=0)
11.26.07.READING INPUT
11.26.17.UPDATE COMPLETE
11.26.18.RETURN(SAMCEM)
11.26.18.RFL(70000)
11.29.03.RUN(A...COMPILE.CLIST.OC)
11.30.16.CTIME 029.123 SEC.  FUN 60 (027-000)
11.30.16.RFL(130000)
11.40.28.PRESET.
11.40.28.OC(LC=377777)
11.41.31. EXECUTION FLD LNGTH  077200
11.42.26.STOP
11.42.27.CP  082.548 SEC.
11.42.28.PP  073.857 SEC.
```

REFERENCES

1. Eisen, Dennis, and Alderson, Linda, "Advanced Bomber Pre-Launch Survivability," Lamda Corporation, Arlington, Virginia, June 1972.

2. Everett, Hugh, III, "Generalized Lagrange Multiplier Method for Solving Problems of Optimum Allocation of Resources," Operations Research, V. 11, pp. 399-417, 1963.

3. Hiller, F. S. and Lieberman, G. J., Introduction to Operations Research, Holden-Day, Inc., September 1968.

4. Thomas, M. A. and Gemmill, G. W., Increasing Salvo Kill Probability Through Aim Point Patterning, US Naval Weapons Laboratory, Technical Report (NWL) TR-2756, Dahlgren, Virginia, June 1972.